MATHEMATICS

Standard VII

Untouchability is a sin

Untouchability is a crime

Untouchability is inhuman

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Chairperson
Tmt. SHEILA BOSE
Principal
Lady Willingdon Institute of Advanced Study in Education
Chennai – 600 005.

Reviewers

Thiru M.K. SUBRAMANIAN
Deputy Director (Retd.)
Directorate of Teacher Education
Research and Training
Chennai-600 006.

Tmt. NIRMALA DEVARAJ
Lecturer in Mathematics (S.G.)
Bharathi Women's College
Chennai-600 108.

Authors

Thiru G. GNANA SUNDARAM
P.G. Teacher
S.S.V. Higher Secondary School
Park Town, Chennai-600 003.

Thiru. M. PALANIVASAN
P.G.Teacher
I.C.F. Higher Secondary School
Chennai-600 038.

Thiru P. NAGARAJAN
B.T. Assistant
B.R.Govt. High School
Puzhal, Thiruvallur Dt.
Chennai-600 066.

Thiru. T. KATHIRVEL
Assistant Headmaster
Govt. Higher Secondary School
Arumbakkam
Chennai-600 106.

Thiru M. SRINIVASAN
B.T. Assistant
Govt. High school
BUDUR, Thiruvallur Dt.
Chennai-600 067

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1. NUMBER WORK

'Numbers and letters are the two eyes of living kind' says Thirukkural. Of these two, numbers are considered as the basis of science. In the development of science, first we should know about the properties and operations on numbers which are very important in our daily life. In the earlier classes we have studied about number system and the fundamental operations on them. Now we extend our study on the number system in this class.

1.1 Integers – Revision
1.2 Rational numbers
1.3 Square roots and cube roots
1.4 Approximations

1.1 Integers – Revision :

1.1.1 Integers and order in integers
1.1.2 Four fundamental operations on integers
1.1.3 Use of brackets and simplification

So far we have studied about natural numbers, whole numbers, integers and fractions and the four fundamental operations on them. We shall revise them briefly in this chapter.

1.1.1 Integers and order in integers :

Natural Numbers :

Counting numbers are called natural numbers.

N = {1, 2, 3, ...} is the set of all natural numbers.

Whole Numbers :

Natural numbers together with 0 are called whole numbers.

W = {0, 1, 2, 3, ...} is the set of all whole numbers.

Integers :

The natural numbers and the negative numbers together with zero are called integers.

Z = {... –3, –2, –1, 0, 1, 2, 3 ... } is the set of all integers.

{1, 2, 3....} is called the set of positive integers.

{... –3, –2, –1} is called the set of negative integers.

{0, 1, 2, 3, ...} is also called the set of non–negative integers.

The integers are represented on the number line as follows:

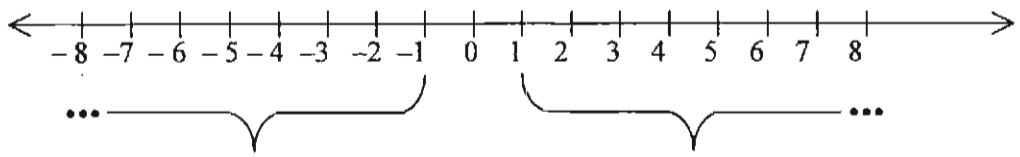


Fig. 1.1

On the number line, the numbers which are to the right of zero are called positive integers. The numbers which are to the left of zero are called negative integers.

Discuss:

1. Is zero a positive number or a negative number?
2. Every natural number is a whole number.
3. Every whole number is an integer.

Order in Integers:

On the number line, we find that the number value increases as we move to the right and decreases as we move to the left.

If we represent two integers on the number line, the integer on the right is greater than the integer on the left. In other words, the integer on the left is lesser than the integer on the right.

For example, consider the following points marked on the number line in the figure given below:

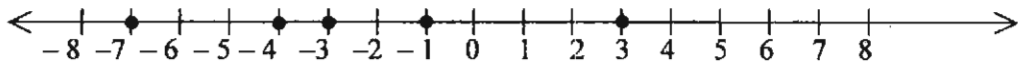


Fig. 1.2

In the above figure,

3 is to the right of –1	$\therefore 3 > -1$
–1 is to the right of – 4	$\therefore -1 > -4$
–3 is to the left of – 1	$\therefore -3 < -1$
–7 is to the left of – 4	$\therefore -7 < -4$

Discuss: Can you find the order in integers without number line? How?

Example 1 :

Which is smaller? – 2 or – 5

Solution :

Mark the integers – 2 and – 5 on the numberline.

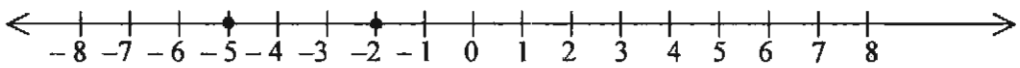


Fig. 1.3

On the numberline , – 5 is on the left side of – 2

$\therefore$ – 5 is smaller than – 2

That is $-5 < -2$

Example 2 :

Write the following integers in ascending order and in descending order.

3 , – 2 , 0 , – 4 , – 1 , 5

Solution :

Mark these integers on the numberline.

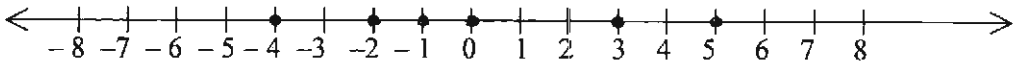


Fig. 1.4

Now arrange the integers from left to right to get them in ascending order

$-4 < -2 < -1 < 0 < 3 < 5$

$\therefore$ The ascending order is $-4, -2, -1, 0, 3, 5$

Arrange the integers from right to left to get them in descending order

$5 > 3 > 0 > -1 > -2 > -4$

$\therefore$ The descending order is $5, 3, 0, -1, -2, -4$.

Exercise 1.1

1. ***Find all the integers between***

   (a) 0 and 5 ***(b) – 1 and 4*** ***(c) – 4 and 0*** ***(d) – 6 and – 2***

2. ***In the following pairs of integers, which one is greater?***

   (a) –15 , – 5 ***(b) 18 , 0*** ***(c) – 33 , – 48*** ***(d) – 50, 0***

3. ***Write the following integers in ascending order***

   (a) 2 , – 3 , – 1 , 4 , – 5 , 0 ***(b) 6 , – 5 , 5 , – 7 , 4 , – 3***

4. ***Write the following in descending order***

   (a) 1 , 0 , 2 , – 3 , 4 , – 7 ***(b) 8 , 0 , – 10 , 6 , – 6 , 5***

1.1.2 Four fundamental operations on integers :

(a) Addition of integers :

Rules: (1) The sum of two positive integers is a positive integer obtained by adding the two integers.

For example, $(+5) + (+7) = +12$

(2) The sum of two negative integers is a negative integer obtained by adding the two integers leaving the sign of the integers and then put the negative sign to the sum.

For example, $(-3) + (-7) = -(3+7) = -10$

(3) To add a positive integer and a negative integer, subtract the smaller number from the greater number without taking the sign into account and to the result put the sign of the greater number.

For example ,

(a) $(+10) + (-4) = +(10-4) = +6$

(b) $(-8) + (+3) = -(8-3) = -5$

(c) $(+5) + (-5) = (5-5) = 0$

Note: If we add a positive number and a negative number sometimes we get zero.

Then these numbers are called the additive inverses of each other.

For example, + 3 and – 3 are the additive inverses of each other.

(b) Subtraction of integers :

To subtract an integer from another integer, add the additive inverse of the second number to the first number.

For example,

(a) $(+5)-(+3)=5+(-3)=5-3=2$

(b) $(+6)-(-2)=6+(+2)=6+2=8$

(c) $(-8)-(+5)=(-8)+(-5)=-(8+5)=-13$

(d) $(-20)-(-5)=(-20)+(+5)=-20+5=-15$

(c) Multiplication of integers :

Rules: (1) The product of two positive integers is a positive integer.

(2) The product of two negative integers is a positive integer.

(3) The product of a positive integer and a negative integer is a negative integer.

For example,

(a) $(+5)\times(+8)=+(5\times8)=+40=40$

(b) $(-5)\times(-8)=+(5\times8)=+40=40$

(c) $(-15)\times(+3)=-(15\times3)=-45$

(d) $(+12)\times(-4)=-(12\times4)=-48$

(d) Division of integers :

The division of two integers need not be an integer. Therefore we can state the rules as follows:

$\frac{\text{Positive Integer}}{\text{Positive Integer}}$ = Positive number $\qquad$ $\frac{\text{Positive Integer}}{\text{Negative Integer}}$ = Negative number

$\frac{\text{Negative Integer}}{\text{Negative Integer}}$ = Positive number $\qquad$ $\frac{\text{Negative Integer}}{\text{Positive Integer}}$ = Negative number

For example,

(a) $\frac{+10}{+5}=+2$ $\qquad$ (b) $\frac{-20}{-5}=+\left(\frac{20}{5}\right)=+4$

(c) $\frac{+60}{-12}=-\left(\frac{60}{12}\right)=-5$ $\qquad$ (d) $\frac{-36}{+9}=-\left(\frac{36}{9}\right)=-4$

Example 3 :

Add the following integers:

(a) + 13 and + 27 $\qquad$ (b) –18 and – 34 $\qquad$ (c) + 65 and – 45

(d) – 89 and + 54 $\qquad$ (e) + 72 , + 12 and –16

Solution :

(a) $(+13)+(+27)=+(13+27)=+40$

(b) $(-18)+(-34)=-(18+34)=-52$

(c) $(+65) + (-45) = +(65-45) = +20$

(d) $(-89) + (+54) = -(89-54) = -35$

(e) $(+72) + (+12) + (-16) = (+84) + (-16) = +(84-16) = +68$

Example 4 :

Subtract the following :

(a) – 562 from 328 (b) – 460 from – 240

(c) 633 from 300 (d) – 128 from 0

Solution :

Subtract the following :

(a) $(328) - (-562) = 328 + (+562) = 328 + 562 = 890$

(b) $(-240) - (-460) = -240 + (+460) = 460 - 240 = 220$

(c) $(300) - (+633) = 300 + (-633) = -(633-300) = -333$

(d) $0 - (-128) = 0 + (+128) = 0 + 128 = 128$

Example 5 :

Find the product of the following:

(a) $(-18) \times (+6)$ (b) $(-25) \times (-8)$ (c) $(7) \times (-8) \times (12)$

(d) $(-15) \times (0) \times (-34)$

Solution :

Find the product of the following :

(a) $(-18) \times (+6) = -(18 \times 6) = -108$

(b) $(-25) \times (-8) = +(25 \times 8) = +200$

(c) $(7) \times (-8) \times (+12) = (-56) \times (+12) = -(56 \times 12) = -672$

(d) $(15) \times (0) \times (-34) = (0) \times (-34) = 0$

Example 6 :

Divide the following numbers :

(a) Divide + 250 by + 50 (b) Divide 81 by (–27)

(c) $(-252) \div (+12)$ (d) $(-111) \div (-37)$

Solution :

(a) Divide $(+250)$ by $(+50)$

$$\frac{+250}{+50} = +\left(\frac{250}{50}\right) = +5$$

(b) Divide 81 by (-27)

$$\frac{+81}{-27} = -\left(\frac{81}{27}\right) = -3$$

(c) $(-252) \div (+12)$

$$\frac{-252}{+12} = -\left(\frac{252}{12}\right) = -21$$

(d) $(-111) \div (-37)$

$$\frac{-111}{-37} = +\left(\frac{111}{37}\right) = +3$$

Example 7 :

Simplify $\dfrac{(-80) \times (+3)}{(-2)^2}$

Solution :

$$\frac{(-80) \times (+3)}{(-2)^2} = \frac{-(80 \times 3)}{(-2) \times (-2)} = \frac{-(80 \times 3)}{+(2 \times 2)} = -\frac{240}{4} = -60$$

Exercise 1.2

1. ***Add:***
   (a) $432 + (-584)$ (b) $(-153) + (-437)$ (c) $(-12) + (-15) + 17$
   (d) $45 + 32 + (-59)$ (e) $55 + (-32) + 38 + (-47)$
   (f) $(-15) + (-10) + 20 + (-25)$
2. ***Subtract:***
   (a) 346 from 286 (b) (-95) from (-100) (c) (-57) from 0
   (d) -186 from 285
3. ***Multiply:***
   (a) (-10) and $(+14)$ (b) (-12) and (-15) (c) (-25) and $(+32)$
   (d) 122 and (-11)
4. ***Divide:***
   (a) 48 by (-6) (b) (-256) by $(+8)$ (c) (-729) by (-81)
   (d) 441 by (-21)
5. ***Simplify:***
   (a) $\dfrac{(15) \times (18)}{(-9)}$ ***(b)*** $\dfrac{16 \times (-9) \times (-2)}{(-3) \times (+8)}$ ***(c)*** $\dfrac{(-20) \times (-12) \times (+25)}{(-5) \times (-4)}$
6. ***Fill in the blanks:***
   (a) To subtract -5 from 15, we add _____ to 15
   (b) The product of a negative integer and zero is _____
   (c) $(-15) +$ _____ $= 0$
   (d) _____ $\times (-14) = +70$
   (e) $(-72) \div$ _____ $= +6$

1.1.3 Use of brackets and simplification :

Arithmetic expression:

A number or combination of numbers formed using the arithmetic operations is called a numerical expression or arithmetic expression.

For example ,

(a) $5 \times 3 + 25$

(b) $2 \times 6 - 5 \times 2$

(c) $30 \div 5 \times 2$

are some examples of arithmetic expressions.

Use of brackets in arithmetic expressions :

When we use two or more fundamental operations in any expression, we get different answers when the order of operation changes.

For example the calculation of $48 \div 12 \times 2$ can be done in two different ways.

One way of calculation is

$$48 \div 12 \times 2 = (48 \div 12) \times 2$$
$$= 4 \times 2$$
$$= 8$$

Another way of calculation is

$$48 \div 12 \times 2 = 48 \div (12 \times 2)$$
$$= 48 \div 24$$
$$= 2$$

From the above two examples, we find that the answers are different as the order of the operation changes. Hence brackets are used to get the same answer for the expression when two or more fundamental operations are involved. Whenever brackets are there in any calculation, the operation inside the bracket is done first and then the operation outside the bracket .

The symbol ( ) is called the bracket or parenthesis. The symbol '(' is called the opening bracket. The symbol ') ' is called the closing bracket.

In longer expressions, we use more than one bracket. The other symbols for brackets are as follows:

{ } are called braces or flower brackets.

[ ] are called square brackets.

Rules for removing the brackets :

While removing the brackets we follow the methods given below:

(1) If there is a number before a bracket, we simplify the expression inside the bracket first and then multiply the number outside the bracket.

For example, $3\ (5 + 8) = 3 \times (5 + 8)$
$= 3 \times 13$
$= 39$

(2) If there is a minus sign before a bracket then the sign of the numbers inside the brackets is changed from 'plus' to 'minus' and from 'minus' to 'plus'.

For example ,

(a) $-(+3 + 5 - 8) = -3 - 5 + 8$

(b) $-(a - b + c) = -a + b - c$

Example 8 :

Simplify the following :

(a) $8 + (5 - 2)$ (b) $5 - (2 + 6)$ (c) $(5 + 8) \times 3$ (d) $(12 + 3) \div 5$

Solution :

Simplify :

(a) $8 + (5 - 2) = 8 + 3$
$= 11$

(b) $5 - (2 + 6) = 5 - 8$
$= -3$

(c) $(5 + 8) \times 3 = 13 \times 3$
$= 39$

(d) $(12 + 3) \div 5 = 15 \div 5$
$= 3$

Example 9 :

Simplify :

$4 + (8 - 2) + (7 - 3)$

Solution :

$4 + (8 - 2) + (7 - 3) = 4 + 6 + 4$
$= 14$

Example 10 :

Simplify :

$11 (12 - 4) - 5 (3 + 17)$

Solution :

$11 (12 - 4) - 5 (3 + 17) = 11 \times (12 - 4) - 5 \times (3 + 17)$
$= 11 \times 8 - 5 \times 20$
$= 88 - 100$
$= -12$

Example 11 :

Simplify:

$75 \div (5 \times 3)$

Solution :

$75 \div (5 \times 3) = 75 \div 15$
$= 5$

Exercise 1.3

Simplify the following expressions :

1. $2 \times (15 - 5)$
2. $(-1)(12 - 7)$
3. $7 - (13 - 2)$
4. $32 - (6 \times 5) + 8$
5. $3 \times (8 + 3) + 2 \times (5 + 2)$
6. $(15 + 25) \times (45 - 25)$
7. $(16 - 4) \times (25 - 17)$
8. $(6 - 16) \times (5 - 11)$
9. $(-49 \div 7) + (-96 \div 8)$
10. $(15 \times 7) \div (35 \times 3)$

THINGS TO REMEMBER

1. **Natural Numbers N = {1, 2, 3, 4, ...}**
2. **Whole numbers W = {0, 1, 2, 3, ...}**
3. **Integers Z = { ... –3, –2, –1, 0, 1, 2, 3, ...}**
4. **If we represent two integers on the numberline, the integer on the right is greater than the integer on the left.**
5. **The sum of two positive integers is a positive integer obtained by adding the two integers.**
6. **The sum of two negative integers is a negative integer obtained by adding the two integers leaving the signs of the integers and then putting the negative sign to the sum.**
7. **To add a positive integer and a negative integer, subtract the smaller number from the greater number without taking the sign into account and to the result put the sign of the greater number.**
8. **To subtract an integer from another integer, add the additive inverse of the second number to the first number.**
9. **(a) The product of two positive integers is a positive integer.**
   (b) The product of two negative integers is a positive integer.
   (c) The product of a positive integer and a negative integer is a negative integer.
10. $\frac{\text{Positive Integer}}{\text{Positive Integer}}$ **= Positive number**

    $\frac{\text{Negative Integer}}{\text{Negative Integer}}$ **= Positive number**

    $\frac{\text{Positive Integer}}{\text{Negative Integer}}$ **= Negative number**

    $\frac{\text{Negative Integer}}{\text{Positive Integer}}$ **= Negative number**
11. **Brackets are the symbols used to simplify the mathematical expressions.**
12. **While removing the brackets we follow the methods given below:**
    If there is a number before the bracket, we simplify the expression inside the bracket first and then multiply the number outside the bracket. If there is a minus sign before a bracket, the sign of the numbers inside the brackets is changed from 'plus' to 'minus' and from 'minus' to 'plus'.

1.2 Rational numbers :

1.2.1	**Fractions and their properties – Revision**
1.2.2	**Rational numbers**
1.2.3	**Comparison of rational numbers**
1.2.4	**Four fundamental operations on rational numbers**

In this chapter we extend the set of all integers to the set of all rational numbers. Before discussing the set of rational numbers let us revise our previous knowledge about fractions.

1.2.1 Fractions and their properties – Revision :

(1) Fraction is a part or parts of a whole. $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{5}$, $\frac{5}{2}$ are all fractions. Any fraction can be represented on a numberline.

(2) In a fraction if the numerator is less than the denominator, the fraction is called a proper fraction. The fractions $\frac{3}{5}$, $\frac{2}{7}$, $\frac{3}{8}$, $\frac{21}{47}$ are all proper fractions.

(3) In a fraction if the numerator is greater than the denominator, the fraction is called an improper fraction. The fractions $\frac{7}{2}$, $\frac{8}{5}$, $\frac{15}{13}$, $\frac{100}{7}$ are all improper fractions.

(4) A number consisting of a natural number and a proper fraction is called a mixed fraction. The numbers $2\frac{1}{4}$, $3\frac{1}{2}$, $8\frac{3}{5}$, $15\frac{2}{9}$ are all mixed fractions.

Note : We can convert the mixed fractions to improper fractions. Similarly we can convert improper fractions to mixed fractions.

(5) Fractions having the same denominator are called like fractions. The fractions $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$ are all like fractions.

(6) Fractions having different denominators are called unlike fractions. The fractions $\frac{2}{3}$, $\frac{5}{7}$, $\frac{4}{9}$, $\frac{10}{17}$ are all unlike fractions.

Note: We can convert the unlike fractions to like fractions.

(7) When two or more fractions represent the same part of the whole, the fractions are called equivalent fractions. The fractions $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$ are called equivalent fractions.

(8) To find an equivalent fraction, both the numerator and the denominator of a fraction are multiplied or divided by the same number.

For example,

$$\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8} \quad \cdots(i) \quad \text{and} \quad \frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2} \quad \cdots(ii)$$

From (i) and (ii) we find that $\frac{1}{2}$ and $\frac{4}{8}$ are equivalent fractions.

(9) To reduce a fraction to its lowest form, both the numerator and the denominator should be either divided by all their common factors in succession or divided by their g.c.d. once.

Example 12 :

Reduce $\frac{105}{180}$ to its lowest form.

Method (i)

$$\frac{105}{180} = \frac{105 \div 5}{180 \div 5} = \frac{21}{36} = \frac{21 \div 3}{36 \div 3} = \frac{7}{12}$$

$$\therefore \frac{105}{180} = \frac{7}{12}$$

Method (ii)

To reduce $\frac{105}{180}$ to its lowest form, first find the g.c.d of 105 and 180.

The g.c.d of 105 and 180 is 15 (How ?)

Now divide the numerator and denominator by 15.

$$\therefore \frac{105}{180} = \frac{105 \div 15}{180 \div 15} = \frac{7}{12}$$

(10) To convert unlike fractions to like fractions, first find the l.c.m of the denominators of all the fractions. Then multiply the numerator and denominator by suitable numbers to get the denominators of the given fractions equal to the l.c.m of the denominators.

Example 13 :

Convert the following unlike fractions into like fractions

$\frac{3}{4}, \frac{5}{6}, \frac{3}{8}$

Solution :

In the fractions $\frac{3}{4}, \frac{5}{6}, \frac{3}{8}$ l.c.m. of the denominators 4, 6, 8 = 24

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$
$$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$
$$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

$\frac{18}{24}, \frac{20}{24}, \frac{9}{24}$ are equivalent like fractions of $\frac{3}{4}, \frac{5}{6}, \frac{3}{8}$ respectively.

(11) For comparison of two or more fractions, we convert them into like fractions and then the fraction which has the greatest numerator is said to be greater than the others.

Example 14 :

Find the greater among $\frac{3}{5}$ and $\frac{5}{8}$

Solution :

First find the l.c.m. of 5 and 8.

The l.c.m. of 5 and 8 is 40

$\frac{3}{5} = \frac{3 \times 8}{5 \times 8} = \frac{24}{40}$ and $\frac{5}{8} = \frac{5 \times 5}{8 \times 5} = \frac{25}{40}$

The like fractions of $\frac{3}{5}$ and $\frac{5}{8}$ are $\frac{24}{40}$ and $\frac{25}{40}$.

Here $\frac{25}{40}$ is greater than $\frac{24}{40}$

$\therefore$ $\frac{5}{8}$ is greater than $\frac{3}{5}$

(12) For the addition of like fractions all the numerators are added and the sum is written as the numerator, keeping the denominator same.

Example 15 :

Add :

$\frac{1}{5} + \frac{2}{5} + \frac{3}{5}$

Solution :

$$\frac{1}{5} + \frac{2}{5} + \frac{3}{5} = \frac{1 + 2 + 3}{5} = \frac{6}{5} = 1\frac{1}{5}$$

(13) For the addition of unlike fractions, first convert them into like fractions using l.c.m and then add the fractions.

Example 16 :

Add :

$$\frac{2}{5} + \frac{3}{8} + \frac{5}{10}$$

Solution:

The l.c.m of the denominators 5, 8, 10 is 40

$$\therefore \frac{2}{5} = \frac{2 \times 8}{5 \times 8} = \frac{16}{40}\ ;\ \frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40} \text{ and } \frac{5}{10} = \frac{5 \times 4}{10 \times 4} = \frac{20}{40}$$

$$\therefore \frac{2}{5} + \frac{3}{8} + \frac{5}{10} = \frac{16}{40} + \frac{15}{40} + \frac{20}{40} = \frac{51}{40} = 1\frac{11}{40}$$

(14) For the subtraction of two like fractions, we find the difference between the numerators and write the numerator keeping the denominator same.

Example 17 :

Subtract $\frac{13}{24}$ from $\frac{23}{24}$

Solution :

$$\frac{23}{24} - \frac{13}{24} = \frac{23 - 13}{24} = \frac{10}{24} = \frac{5}{12}$$

(15) For the subtraction of two unlike fractions change the fractions to like fractions and do the subtraction.

Example18 :

Subtract $\frac{1}{10}$ from $\frac{1}{4}$

Solution :

Find $\frac{1}{4} - \frac{1}{10}$ .L.c.m. of the denominators 4 , 10 is 20

$$\frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20} \text{ and } \frac{1}{10} = \frac{1 \times 2}{10 \times 2} = \frac{2}{20}$$

$$\therefore \frac{1}{4} - \frac{1}{10} = \frac{5}{20} - \frac{2}{20} = \frac{3}{20}$$

(16) To find the product of two fractions, multiply their numerators and their denominators separately and put them as a new fraction.

Example 19 :

Find the product of $1\frac{3}{4}$ and $3\frac{1}{7}$

Solution :

$$1\frac{3}{4} \times 3\frac{1}{7} = \frac{7}{4} \times \frac{22}{7} = \frac{7 \times 22}{4 \times 7} = \frac{154}{28} = \frac{22}{4} = \frac{11}{2} = 5\frac{1}{2}$$

(17) To divide a fraction by another fraction, multiply the first fraction by the reciprocal of the second fraction.

Example 20 :

Find $\frac{9}{22} \div \frac{5}{33}$

Solution : $\frac{9}{22} \div \frac{5}{33} = \frac{9}{22} \times \frac{33}{5} = \frac{9 \times 33}{22 \times 5} = \frac{297}{110} = 2\frac{7}{10}$

Exercise 1.4

1. (a) Which is greater? $\frac{3}{8}$ *or* $\frac{4}{7}$

(b) Which is smaller? $\frac{7}{12}$ *or* $\frac{8}{15}$

2. Simplify the fractions to the lowest form :

(a) $\frac{75}{125}$ *(b)* $\frac{27}{81}$ *(c)* $\frac{100}{60}$ *(d)* $\frac{14}{126}$

3. Add :

(a) $\frac{5}{9} + \frac{7}{18}$ *(b)* $\frac{12}{15} + \frac{11}{20}$ *(c)* $2\frac{1}{3} + 3\frac{8}{9}$ *(d)* $7 + \frac{6}{11}$

4. Subtract :

(a) $\frac{8}{13}$ *from* $\frac{15}{26}$ *(b)* $\frac{2}{3}$ *from* $\frac{7}{8}$

5. Simplify :

(a) $4 - 3\frac{1}{5}$ *(b)* $5\frac{2}{3} - 2\frac{3}{4}$

6. Simplify:

(a) $\frac{3}{4} + \frac{5}{8} - \frac{7}{12}$ *(b)* $\frac{1}{6} + 2\frac{1}{2} - 1\frac{1}{3}$ *(c)* $\frac{1}{2} - \left(\frac{1}{3} - \frac{1}{4}\right)$

(d) $\frac{7}{8} + \left(\frac{1}{3} - \frac{1}{5}\right)$

7. Simplify :

(a) $\frac{5}{22} \times \frac{66}{125}$ *(b)* $2\frac{5}{8} \times 2\frac{2}{7}$

(c) $\frac{9}{20} \times \frac{3}{7} \times \frac{14}{27}$ *(d)* $3\frac{1}{2} \times 7\frac{1}{3} \times 1\frac{2}{7}$

8. Simplify :

(a) $\frac{5}{6} \div \frac{10}{3}$ *(b)* $20 \div \frac{2}{3}$ *(c)* $\frac{15}{19} \div 5$ *(d)* $6\frac{2}{3} \div \frac{4}{21}$

1.2.2 Rational numbers :

Rational numbers:

We have learnt about integers. We know that sum and difference of two integers is an integer. Multiplication of two integers is also an integer. But, when we divide an integer by another integer, do we always get an integer?

For example $\frac{5}{7}$, $\frac{-3}{8}$, $\frac{-9}{7}$, $\frac{6}{-5}$, $\frac{2}{5}$ are all not integers.

From the above examples we notice that when an integer is divided by another integer, the result may not be an integer. In such cases, we need an extended system of numbers which we call as rational numbers.

What is a rational number ?

A rational number is the ratio of two integers and is of the form $\frac{p}{q}$ where both p and q are integers and $q \neq 0$.

Discuss :

(a) Correlate the word ratio with rational numbers.

(b) Think what will happen if $q = 0$ in $\frac{p}{q}$

(c) Rational numbers are of greater importance in real life situations.

Note : (1). All fractions are rational numbers.

(2). All integers are rational numbers.

The rational number $\frac{8}{1}$ is the same as the integer 8.

The rational number $\frac{-5}{1}$ is the same as the integer –5.

The rational number $\frac{p}{q}$ when $q = 1$ is considered to be the same as integer p.

Thus rational numbers include all the integers.

Positive and negative rational numbers :

Rational numbers can be classified as positive rational numbers, zero and negative rational numbers.

The numbers $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{2}$ ... are all examples of positive rational numbers.

The numbers $\frac{-1}{2}$, $\frac{-4}{5}$, $\frac{6}{-7}$, $\frac{-5}{3}$ ... are all examples of negative rational numbers.

Note: (a) For any negative rational number either the numerator or the denominator should be a negative integer.

(b) If the numerator and the denominator both are negative integers then it is a positive rational number.

Standard form of a rational number :

Recall that every fraction can be expressed in its lowest terms by dividing the numerator and the denominator by their g.c.d.

In a similar manner we can express every rational number in a standard form so that

(a) its denominator is always positive

(b) the numerator and the denominator have no common divisors other than 1.

That is a rational number $\frac{p}{q}$ is said to be in its standard form, if q is positive and the integers p and q have no common divisors other than 1.

Note : $\frac{+a}{-b}, \frac{-a}{+b}, -\frac{a}{b}$ all represent the same rational number.

Example 21 :

Write the following integers as rational numbers :

$5, -8, -13, 27, -40, 0$

Solution :

The above integers are expressed in the form $\frac{p}{q}$ as follows

$$\frac{5}{1}, \frac{-8}{1}, \frac{-13}{1}, \frac{27}{1}, \frac{-40}{1}, \frac{0}{1}$$

Therefore integers can also be called as rational numbers.

Example 22 :

Write the following rational numbers as integers :

$$\frac{11}{1}, \frac{-17}{1}, \frac{10}{1}, \frac{0}{1}, \frac{-4}{1}$$

Solution :

The above rational numbers can be written as 11, –17, 10, 0, – 4 which are all integers.

Example 23 :

Express $\frac{-3}{5}$ as a rational number whose

(a) numerator is – 9 (b) denominator is –10 (c) denominator is –35

Solution :

(a) To get – 9 in the numerator, the numerator (– 3) should be multiplied by 3.

∴ Multiply the numerator and denominator by 3.

$$\frac{-3}{5} = \frac{(-3) \times 3}{5 \times 3} = \frac{-9}{15}$$

(b) To get –10 in the denominator, the denominator 5 should be multiplied by (– 2)

∴ Multiply the numerator and denominator by (– 2)

$$\frac{-3}{5} = \frac{(-3) \times (-2)}{(5) \times (-2)} = \frac{+6}{-10}$$

(c) To get – 35 in the denominator, the denominator should be multiplied by (– 7)

∴ Multiply the numerator and denominator by (– 7)

$$\frac{-3}{5} = \frac{(-3) \times (-7)}{(5) \times (-7)} = \frac{+21}{-35}$$

Example 24 :

Express the rational numbers $\frac{4}{-5}, \frac{-3}{-2}, \frac{+8}{-11}$ with positive denominators.

Solution :

To express the above rational numbers with positive denominators multiply both numerator and denominator by (– 1).

$$\therefore \frac{4}{-5} = \frac{4 \times (-1)}{(-5) \times (-1)} = \frac{-4}{+5}$$

$$\frac{-3}{-2} = \frac{(-3) \times (-1)}{(-2) \times (-1)} = \frac{+3}{+2} = \frac{3}{2}$$

$$\frac{+8}{-11} = \frac{(+8) \times (-1)}{(-11) \times (-1)} = \frac{-8}{+11} = -\frac{8}{11}$$

Example 25 :

Express the following rational numbers in standard form

(a). $\frac{-6}{24}$ (b) $\frac{+35}{-85}$ (c) $\frac{-18}{-96}$

Solution :

(a) The denominator of $\frac{-6}{24}$ is positive

The g.c.d of 6 and 24 is 6.

∴ Divide both numerator and denominator by 6.

$$\therefore \frac{(-6) \div 6}{24 \div 6} = \frac{-1}{4} = -\frac{1}{4}$$

The standard form of $\frac{-6}{24} = -\frac{1}{4}$

(b) The denominator of $\frac{+35}{-85}$ is negative

To convert the negative denominator into a positive denominator, multiply the numerator and the denominator by (–1).

$$\therefore \frac{+35}{-85} = \frac{(+35) \times (-1)}{(-85) \times (-1)} = \frac{-35}{+85}$$

The g.c.d of 35 and 85 is 5 (How ?)

Dividing the numerator and denominator by 5 we get

$$\frac{(-35) \div 5}{85 \div 5} = \frac{-7}{17} = -\frac{7}{17}$$

$$\therefore \text{ The standard form of } \frac{+35}{-85} = -\frac{7}{17}$$

(c) The denominator of $\frac{-18}{-96}$ is negative. Therefore first convert the negative denominator into positive denominator.

$$\therefore \frac{-18}{-96} = \frac{(-18) \times (-1)}{(-96) \times (-1)} = \frac{18}{96}$$

The g.c.d of 18 and 96 is 6

$$\therefore \frac{18 \div 6}{96 \div 6} = \frac{3}{16}$$

$$\therefore \text{The standard form of } \frac{-18}{-96} = \frac{3}{16}$$

1.2.3 Comparison of rational numbers :

We know that fractions and integers have order relations. In the same way rational numbers also have an order relation.

To compare the rational numbers we use the order relation.

We can compare the rational numbers in two ways.

(1). Using the numberline.

(2). Using the arithmetical process

(1) Comparison of rational numbers using the numberline:

To compare the rational numbers using the numberline , the rational numbers should have the same denominator. Arrange these rational numbers on the numberline in the ascending order from left to right and compare.

(i) Suppose the rational numbers are having 3 as denominator, we represent these numbers on the numberline as shown below:

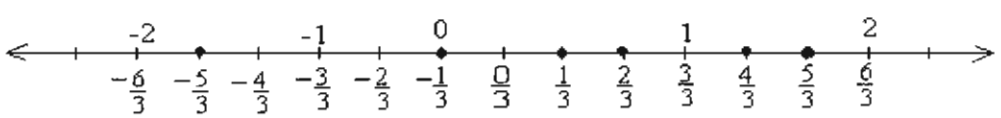


Fig. 1.5

From the figure, we find the relation between two numbers as follows:

(a) $\frac{2}{3} > \frac{1}{3}$ (b) $\frac{5}{3} > \frac{4}{3}$ (c) $\frac{1}{3} > \frac{-1}{3}$ (d) $\frac{-1}{3} > \frac{-5}{3}$

(ii) Suppose the rational numbers are having 4 as the denominator, we can represent the numbers on the number line as follows:

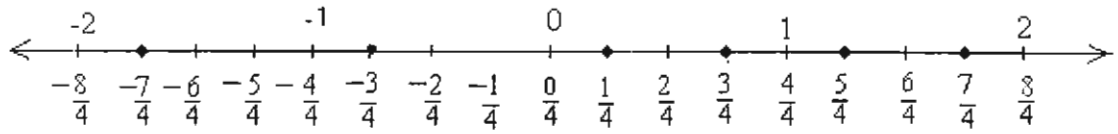


Fig. 1.6

From the above numberline we get the following order relations:

(a) $\frac{7}{4} > \frac{3}{4}$ (b) $\frac{5}{4} > \frac{1}{4}$ (c) $\frac{1}{4} > \frac{-3}{4}$ (d) $\frac{-3}{4} > \frac{-7}{4}$

Note: To compare rational numbers using numberline, all the rational numbers should have the same denominator.

(2) Comparison of rational numbers using arithmetical process :

(a) Rational numbers having the same denominator :

If the denominators of two rational numbers are the same, use the order relation among integers for the numerator and compare.

Example 26 :

Compare $\frac{-4}{9}$ and $\frac{-7}{9}$

Solution :

Here the denominators are the same. Now compare the numerators -4 and -7.

Here $-4 > -7$ (from integer order relation)

Therefore $\frac{-4}{9} > \frac{-7}{9}$

(b) Rational numbers having different denominators :

If the denominators of two rational numbers are not the same, first we should change them into rational numbers having the same denominator and then compare.

Example 27 :

Compare $\frac{-7}{8}$ and $\frac{-5}{6}$

Solution :

First make the denominator of the rational numbers equal.

$$\frac{-7}{8} = \frac{(-7) \times 3}{8 \times 3} = \frac{-21}{24}$$

$$\frac{-5}{6} = \frac{(-5) \times 4}{6 \times 4} = \frac{-20}{24}$$

Now the denominators are the same and $-21 < -20$

$$\therefore \frac{-21}{24} < \frac{-20}{24}$$

$$\text{(i.e) } \frac{-7}{8} < \frac{-5}{6}$$

Exercise 1.5

1. *Express $\frac{-4}{7}$ as a rational number whose*

   (a) numerator is -8 *(b) denominator is 21*

2. *Express the following rational numbers with positive denominators :*

   (a) $\frac{+10}{-15}$ *(b)* $\frac{-3}{-13}$ *(c)* $\frac{+9}{-4}$ *(d)* $\frac{-1}{-4}$

3. *Express the following rational numbers in the standard form :*

   (a) $\frac{-12}{+16}$ *(b)* $\frac{-18}{24}$ *(c)* $\frac{21}{-35}$ *(d)* $\frac{-70}{+42}$

4. *Write the following numbers in the standard form of rational numbers*

   (a) $1\frac{2}{19}$ *(b)* $-2\frac{1}{6}$ *(c)* $-3\frac{2}{5}$ *(d)* $-15\frac{4}{7}$

5. *Write the following as mixed numbers*

   (a) $\frac{25}{8}$ *(b)* $\frac{-19}{5}$ *(c)* $\frac{-29}{12}$ *(d)* $\frac{-23}{-6}$

6. *Fill in the boxes:*

   (a) $-\frac{2}{3} = \frac{\square}{9} = \frac{\square}{-15}$ *(b)* $\frac{-3}{-8} = \frac{+6}{\square} = \frac{\square}{24}$

   (c) $\frac{-4}{13} = \frac{\square}{39} = \frac{20}{\square}$ *(d)* $\frac{-6}{\square} = \frac{3}{5} = \frac{-12}{\square}$

7. *Convert the following rational numbers to the standard form and compare*

   (a) $\frac{-15}{20}$ *and* $\frac{25}{-30}$ *(b)* $\frac{-7}{21}$ *and* $\frac{3}{-9}$ *(c)* $\frac{-8}{-14}$ *and* $\frac{13}{21}$ *(d)* $\frac{-6}{10}$ *and* $\frac{11}{-15}$

8. *Which of the two rational numbers is greater?*

   (a) $\frac{-4}{7}, \frac{5}{-7}$ *(b)* $\frac{-5}{8}, \frac{-3}{4}$ *(c)* $\frac{5}{-8}, \frac{-7}{12}$ *(d)* $\frac{4}{-3}, \frac{-8}{5}$

1.2.4 Four fundamental operations on rational numbers :

The four fundamental operations on rational numbers are similar to that of operations on integers and fractions.

(a) Addition of rational numbers :

(1) For addition of rational numbers with the same denominator, all the numerators are added and the sum is written as the numerator in the result, keeping the denominator same.

(2) For addition of rational numbers with different denominators

(i) find l.c.m of all the denominators

(ii) convert each of the given rational number into an equivalent rational number with the same l.c.m. as denominator.

(iii) taking the l.c.m as the common denominator add all the numerators.

Example 28 :

(a) Add $\frac{3}{8}$ and $\frac{-5}{8}$

(b) Add $\frac{-2}{11}, \frac{3}{11}$ and $\frac{-4}{11}$

Solution :

(a) Add $\frac{3}{8}$ and $\frac{-5}{8}$

$$\frac{3}{8} + \left(\frac{-5}{8}\right) = \frac{3 - 5}{8} = \frac{-2}{8} = \frac{-1}{4}$$

(b) Add $\frac{-2}{11}, \frac{3}{11}$ and $\frac{-4}{11}$

$$\frac{-2}{11} + \frac{3}{11} + \frac{-4}{11} = \frac{-2 + 3 - 4}{11} = \frac{-3}{11}$$

Example 29 :

(a) Add $\left(\frac{-4}{9}\right)$ and $\frac{5}{18}$

(b) Find $\frac{-3}{4} + \frac{-1}{2} + \frac{5}{6}$

Solution :

(a) $\left(\frac{-4}{9}\right) + \frac{5}{18}$

l.c.m. of 9 and 18 is 18

$$\therefore \frac{-4}{9} = \frac{-4 \times 2}{9 \times 2} = \frac{-8}{18}$$

$$\left(\frac{-4}{9}\right) + \frac{5}{18} = \frac{-8}{18} + \frac{5}{18} = \frac{-8 + 5}{18} = \frac{-3}{18} = -\frac{1}{6}$$

$$\therefore \left(\frac{-4}{9}\right) + \frac{5}{18} = -\frac{1}{6}$$

(b) Find $\frac{-3}{4} + \frac{-1}{2} + \frac{5}{6}$

Solution :

l.c.m of the denominators 4 , 2 and 6 is 12

Convert each rational number with the common denominator 12.

$$\frac{-3}{4} = \frac{-3 \times 3}{4 \times 3} = \frac{-9}{12}$$

$$\frac{-1}{2} = \frac{-1 \times 6}{2 \times 6} = \frac{-6}{12}$$

$$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

$$\therefore \frac{-3}{4} + \frac{-1}{2} + \frac{5}{6} = \left(\frac{-9}{12}\right) + \left(\frac{-6}{12}\right) + \left(\frac{10}{12}\right) = \frac{-9-6+10}{12} = \frac{-15+10}{12} = \frac{-5}{12}$$

(b) Subtraction of rational numbers :

(i) For subtraction of rational numbers with the same denominator, subtract the two numerators and the result is written as numerator, keeping the denominator same.

(ii) For subtraction of rational numbers with different denominators, first find l.c.m of the denominators. Then convert the rational numbers into equivalent rational numbers with the common denominator and then subtract.

Example 30 :

(a) Subtract $\frac{8}{23}$ from $\frac{-12}{23}$ (b) Find $\left(\frac{+6}{35}\right) - \left(\frac{-8}{35}\right)$

Solution :

(a) Subtract $\frac{8}{23}$ from $\frac{-12}{23}$

$$\left(\frac{-12}{23}\right) - \left(\frac{8}{23}\right) = \frac{-12 \ -8}{23} = \frac{-20}{23}$$

(b) Find $\left(\frac{+6}{35}\right) - \left(\frac{-8}{35}\right)$

$$\left(\frac{+6}{35}\right) - \left(\frac{-\ 8}{35}\right) = \frac{6}{35} + \frac{8}{35} = \frac{6\ +\ 8}{35} = \frac{14}{35} = \frac{2}{5}$$

Example 31 :

Find $\left(-1\frac{7}{22}\right) - \left(\frac{-12}{33}\right)$

Solution:

$$\left(-1\frac{7}{22}\right) - \left(\frac{-12}{33}\right) = \left(\frac{-29}{22}\right) - \left(\frac{-12}{33}\right)$$

l.c.m of 22 and 33 is 66

$$\left(\frac{-29}{22}\right) = \frac{-29 \times 3}{22 \times 3} = \frac{-87}{66}$$

$$\left(\frac{-12}{33}\right) = \frac{-12 \times 2}{33 \times 2} = \frac{-24}{66}$$

$$\therefore \left(-1\frac{7}{22}\right) - \left(\frac{-12}{33}\right) = \left(\frac{-87}{66}\right) - \left(\frac{-24}{66}\right)$$

$$= \frac{-87 + 24}{66}$$

$$= \frac{-63}{66}$$

$$= \frac{-21}{22}$$

Example 32 :

The sum of two rational numbers is $\frac{3}{4}$. If one of the numbers is $\frac{-4}{5}$ find the other.

Solution :

To find the other number, subtract the given number from the sum

$$\frac{3}{4} - \left(\frac{-4}{5}\right) = \frac{3}{4} + \frac{4}{5}$$

$$\therefore \frac{3}{4} + \frac{4}{5} = \frac{3 \times 5}{4 \times 5} + \frac{4 \times 4}{5 \times 4}$$

$$= \frac{15}{20} + \frac{16}{20}$$

$$= \frac{31}{20}$$

(c) Product of rational numbers :

To find the product of two rational numbers, multiply the numerators and the denominators separately and put them as a new rational number. Simplify the new rational number into its lowest form.

Example 33 :

Find the product of $\frac{4}{-11}$ and $\left(\frac{-22}{8}\right)$

Solution :

$$\left(\frac{4}{-11}\right) \times \left(\frac{-22}{8}\right) = \left(\frac{-4}{11}\right) \times \left(\frac{-22}{8}\right)$$

$$= \frac{-4 \times (-22)}{11 \times 8}$$

$$= \frac{88}{88}$$

$$= 1$$

Example 34 :

Find the product of $\left(-3\frac{4}{15}\right)$ and $\left(-2\frac{1}{49}\right)$

Solution:

$$\left(-3\frac{4}{15}\right) \times \left(-2\frac{1}{49}\right) = \left(\frac{-49}{15}\right) \times \left(\frac{-99}{49}\right)$$

$$= \frac{-49 \times -99}{15 \times 49}$$

$$= \frac{99}{15}$$

$$= \frac{33}{5}$$

$$= 6\frac{3}{5}$$

(d) Multiplicative inverse of a rational number :

If the product of two rational numbers is equal to 1, then one number is called the multiplicative inverse of the other.

(i) $\frac{7}{23} \times \frac{23}{7} = 1$

$\therefore$ The multiplicative inverse of $\frac{7}{23}$ is $\frac{23}{7}$

(ii) $\left(\frac{-8}{11}\right) \times \left(\frac{11}{-8}\right) = 1$

$\therefore$ The multiplicative inverse of $\frac{-8}{11}$ is $\frac{11}{-8}$

(e) Division of rational numbers :

To divide one rational number by another rational number, multiply the first rational number with the multiplicative inverse of the second rational number.

Example 35 :

Find $\left(\frac{2}{3}\right) \div \left(\frac{-5}{6}\right)$

Solution :

$\left(\frac{2}{3}\right) \div \left(\frac{-5}{6}\right) = \frac{2}{3} \times \frac{6}{-5}$ (The multiplicative inverse of $\left(\frac{-5}{6}\right)$ is $\left(\frac{6}{-5}\right)$)

$= \frac{2}{3} \times \left(\frac{-6}{5}\right)$

$= \frac{-4}{5}$

Example 36 :

Find $4\frac{2}{7} \div 2\frac{1}{7}$

Solution :

$4\frac{2}{7} \div 2\frac{1}{7} = \frac{30}{7} \div \frac{15}{7}$

$= \frac{30}{7} \times \frac{7}{15}$ (The multiplicative inverse of $\frac{15}{7}$ is $\frac{7}{15}$)

$= 2$

Example 37 :

Divide $-2\frac{3}{5}$ by $3\frac{1}{6}$

Solution :

$-2\frac{3}{5} \div 3\frac{1}{6} = \left(\frac{-13}{5}\right) \div \frac{19}{6}$

$= \frac{-13}{5} \times \frac{6}{19}$ (since the multiplicative inverse of $\frac{19}{6}$ is $\frac{6}{19}$)

$= \frac{-78}{95}$

Example 38 :

The product of two rational numbers is $\frac{-4}{7}$. If one of the numbers is $\frac{-9}{7}$, find the other.

Solution:

(Given number) × (the other number) = Product of numbers

$\therefore$ The other number = $\frac{\text{Product of numbers}}{\text{given number}}$

$\therefore$ The other number = $\left(\frac{-4}{7}\right) \div \left(\frac{-9}{7}\right)$

$$= \left(\frac{-4}{7}\right) \times \left(\frac{-7}{9}\right)$$

$$= \frac{4}{9}$$

Exercise 1.6

1. *Add :*

   (a) $\frac{-12}{5}$ and $\frac{7}{5}$ (b) $\frac{-7}{9}$ and $\frac{-5}{9}$ (c) $\frac{+5}{17}$ and $\frac{-15}{17}$ (d) $\frac{-10}{23}$ and $\frac{-13}{23}$

2. *Find the sum of the following :*

   (a) $\frac{-3}{4} + \frac{6}{7}$ (b) $\frac{-6}{13} + \frac{-14}{39}$

   (c) $\frac{11}{39} + \left(\frac{-7}{26}\right)$ (d) $\left(\frac{-2}{5}\right) + \frac{5}{6} + \left(\frac{-7}{10}\right)$

3. *Simplify :*

   (a) $\left(-1\frac{2}{11}\right) + \left(-3\frac{4}{11}\right) + \left(5\frac{3}{11}\right)$ (b) $\left(-3\frac{9}{20}\right) + \left(2\frac{7}{10}\right)$

   (c) $\left(-3\frac{4}{5}\right) + \left(2\frac{3}{8}\right)$ (d) $\left(-1\frac{5}{12}\right) + \left(-2\frac{6}{11}\right)$

4. *Multiply :*

   (a) $\frac{-3}{7}$ and $\frac{7}{8}$ (b) $\frac{-6}{11}$ and $\frac{55}{12}$ (c) $\frac{-50}{7}$ and $\frac{21}{10}$ (d) $\frac{-5}{6}$ and $\frac{4}{-15}$

5. *Find the value of the following :*

   (a) $\frac{9}{5} \times \frac{-10}{3} \times \frac{15}{18}$ (b) $\frac{-8}{25} \times \frac{-5}{16} \times \frac{-30}{12}$

   (c) $-3\frac{4}{15} \times -2\frac{1}{49}$ (d) $1\frac{1}{5} \times 2\frac{3}{5} \times -1\frac{2}{13}$

6. (a) Divide $\frac{-4}{9}$ by $\frac{-16}{3}$ (b) Find $\frac{3}{5} \div \frac{-4}{5}$

   (c) Simplify $\frac{-8}{35} \div \frac{-6}{30}$ (d) Find $-9\frac{3}{4} \div \frac{13}{40}$

7. *The sum of two rational numbers is* $\frac{1}{4}$. *If one of the numbers is* $\frac{-4}{5}$, *find the other number .*

8. *What number should be added to* $\frac{-5}{8}$ *so as to get* $\frac{4}{9}$ *?*

9. *The product of two rational numbers is* $\frac{-8}{13}$. *If one of the numbers is* $\frac{-4}{11}$, *find the other .*

10. *By what number should we multiply* $\frac{-3}{7}$ *so that the product is* $\frac{2}{5}$ *?*

THINGS TO REMEMBER

1. A rational number is the ratio of two integers and is of the form $\frac{p}{q}$ Where both p and q are integers and $q \neq 0$
2. All integers are rational numbers. The rational number $\frac{p}{q}$, when $q = 1$ is considered to be the same as the integer p .
3. We can express every rational number in the standard form so that
   (a) its denominator is always positive.
   (b) The numerator and the denominator have no common divisors other than 1.
4. $\frac{+a}{-b}, \frac{-a}{+b}, -\frac{a}{b}$ all represent the same rational number.
5. To compare the rational numbers using numberline, the rational numbers should have the same denominator. Arrange these rational numbers on the numberline in ascending order from left to right and compare.
6. To compare the rational numbers using arithmetical process, use the following methods:
   (a) For the rational numbers having the same denominator:
   If the denominators of the two rational numbers are the same, use the order relation for integers of the numerators and compare.
   (b) Rational numbers having different denominators:
   If the denominators of two rational numbers are not the same, first we should change them into rational numbers having the same denominators and then compare.
7. For addition of rational numbers with the same denominators, all the numerators are added and the sum is written as the numerator in the result, keeping the denominator same.
8. For addition of rational numbers with different denominators,
   a) Find l.c.m of all the denominators
   b) Convert each of the given rational number into an equivalent rational number with the same l.c.m as denominator.
   c) Taking the l.c.m as the common denominator, add all the numerators.
9. For subtraction of rational numbers with the same denominator subtract the two numerators and the result is written as numerator, keeping the denominator same.

10. **For subtraction of rational numbers with different denominators, first find l.c.m of the denominators. Then convert the rational numbers into equivalent rational numbers with common denominators and then subtract.**
11. **To find the product of two rational numbers, multiply the numerators and denominators separately and put them as a new rational number. Simplify the new rational number into its lowest form.**
12. **To divide one rational number by another rational number, multiply the first rational number with the multiplicative inverse of the second rational number**

1.3 Square roots and cube roots :

1.3.1 Square roots of perfect square numbers
1.3.2 Cube roots of perfect cube numbers

1.3.1 Square roots of perfect square numbers :

Perfect square numbers:

When a number is multiplied by the same number the product is known as the square of that number.

$x \times x$ is called the square of x and is written as x^2.

Here x is called the base and 2 is called the index or the power.

x^2 is read as 'x squared'

Now observe the following

$1^2 = 1 \times 1 = 1$ $\quad\quad$ $6^2 = 6 \times 6 = 36$

$2^2 = 2 \times 2 = 4$ $\quad\quad$ $7^2 = 7 \times 7 = 49$

$3^2 = 3 \times 3 = 9$ $\quad\quad$ $8^2 = 8 \times 8 = 64$

$4^2 = 4 \times 4 = 16$ $\quad\quad$ $9^2 = 9 \times 9 = 81$

$5^2 = 5 \times 5 = 25$ $\quad\quad$ $10^2 = 10 \times 10 = 100$

The numbers 1 , 4 , 9 , 16 , 25 , 36 , 49 , 64 , 81 , 100 are called squares of 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8, 9 , 10 respectively.

These square numbers are also called **perfect squares.**

A number is a **perfect square** if the number can be expressed as the product of two same numbers.

Is 100 a perfect square?

Yes, because 100 can be expressed as the product of two same numbers as 10×10.

Is 50 a perfect square?

No. 50 is not a perfect square since 50 cannot be written as the product of two same numbers.

Square Roots :

If any number is expressed as $x \times x$, then x is called the square root of that number.

We know that $5^2 = 5 \times 5 = 25$

Here 25 is called the square of 5

and 5 is called the square root of 25.

Similarly $7^2 = 7 \times 7 = 49$

Here 49 is called the square of 7 and 7 is called the square root of 49.

$$\left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) = \frac{2 \times 2}{3 \times 3} = \frac{4}{9}$$

$\frac{4}{9}$ is called the square of $\frac{2}{3}$ and $\frac{2}{3}$ is called the square root of $\frac{4}{9}$

$(0.4)^2 = (0.4) \times (0.4) = 0.16$

Here 0.16 is called the square of 0. 4 and 0.4 is called the square root of 0.16

Note: For every two same factors, the square root contains only one factor. That is square root of $a \times a$ is a.

The symbol $\sqrt{\ }$ is used to denote the square root of a number.

Thus $\sqrt{25} = 5$, $\sqrt{49} = 7$, $\sqrt{\frac{4}{9}} = \frac{2}{3}$ and $\sqrt{0.16} = 0.4$

Note: We know that $(+5)^2 = (+5) \times (+5) = 25$

Also $(-5)^2 = (-5) \times (-5) = 25$

In the above, 25 is the square of both $(+5)$ and (-5).

$\therefore$ $(+5)$ and (-5) are the square roots of 25.

We find that there are two square roots $+5$ and -5 for the number 25.

In general we can find two square roots for every number. They are positive square root and negative square root. Here after we deal with only positive square roots.

Note: We write the square root of x by $x^{1/2}$ also.

Thus $4^{1/2} = \sqrt{4} = 2$; $100^{1/2} = \sqrt{100} = 10$

Example 39 :

Find the positive square roots of

(a) 121 (b) $\frac{9}{25}$ (c) 0.25

Solution :

(a) $\sqrt{121} = \sqrt{11 \times 11} = \sqrt{11^2} = 11$

$\therefore$ The positive square root of 121 is 11.

(b) $\sqrt{\frac{9}{25}} = \sqrt{\frac{3 \times 3}{5 \times 5}} = \sqrt{\left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right)} = \sqrt{\left(\frac{3}{5}\right)^2} = \frac{3}{5}$

$\therefore$ The positive square root of $\frac{9}{25}$ is $\frac{3}{5}$

(c) $\sqrt{0.25} = \sqrt{0.5 \times 0.5} = \sqrt{(0.5)^2} = 0.5$

$\therefore$ The positive square root of 0.25 is 0.5

Obtaining the square root by splitting the numbers into prime factors :

We know that any composite number can be expressed as the product of prime factors. Using this method we can easily obtain the square roots of perfect square numbers.

Example 40 :

Find the square root of 144

Solution :

Split the number into the product of prime factors.

$144 = 3 \times 3 \times 2 \times 2 \times 2 \times 2$

$\sqrt{144} = \sqrt{3^2 \times 2^2 \times 2^2}$

$= 3 \times 2 \times 2$

$\therefore \sqrt{144} = 12$

3	144
3	48
2	16
2	8
2	4
2	2
	1

Example 41 :

Find the square root of $5^3 \times 5^5$

Solution :

$5^3 \times 5^5 = (5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5 \times 5)$

$= \underbrace{5 \times 5} \times \underbrace{5 \times 5} \times \underbrace{5 \times 5} \times \underbrace{5 \times 5}$

$= 5^2 \times 5^2 \times 5^2 \times 5^2$

$\therefore \sqrt{5^3 \times 5^5} = \sqrt{5^2 \times 5^2 \times 5^2 \times 5^2}$

$= 5 \times 5 \times 5 \times 5$

$= 625$

$\therefore \sqrt{5^3 \times 5^5} = 625$

Example 42 :

Find the square root of 2025

Solution :

$\sqrt{2025} = \sqrt{5 \times 5 \times 3 \times 3 \times 3 \times 3}$

$= \sqrt{5^2 \times 3^2 \times 3^2}$

$= 5 \times 3 \times 3$

$= 45$

$\therefore \sqrt{2025} = 45$

5	2025
5	405
3	81
3	27
3	9
3	3
	1

Example 43 :

Find the square root of $\frac{49}{225}$

Solution :

First find the prime factors of 49 and 225

Factors of $49 = 7 \times 7 = 7^2$

7 | 49
7 | 7
1

3 | 225
3 | 75
5 | 25
5 | 5
1

Factors of $225 = (3 \times 3) \times (5 \times 5) = 3^2 \times 5^2$

$$\sqrt{\frac{49}{225}} = \frac{\sqrt{49}}{\sqrt{225}} = \frac{\sqrt{7^2}}{\sqrt{3^2 \times 5^2}} = \frac{7}{3 \times 5} = \frac{7}{15}$$

$$\therefore \sqrt{\frac{49}{225}} = \frac{7}{15}$$

Example 44 :

Find the square root of $1\frac{9}{16}$

Solution :

The number $1\frac{9}{16}$ is a mixed number. First convert this into improper fraction.

$$\therefore 1\frac{9}{16} = \frac{16 + 9}{16} = \frac{25}{16}$$

$$\therefore \sqrt{1\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4} \qquad \therefore \sqrt{1\frac{9}{16}} = \frac{5}{4}$$

Note: Using the idea of square and square roots we can find the area of the square and from the area we can find the side of the square.

Discuss:

(1) In square numbers prime factors occur in pairs and the index is even.

(2) Is $\sqrt{3^2 + 4^2} = 3 + 4$? Why?

1.3.2 Cube roots of perfect cube numbers :

Perfect cube numbers:

When a number is multiplied thrice by itself, the product is the cube of that number.

$x \times x \times x$ is called the cube of x and is written as x^3

Now observe the following:

$1^3 = 1 \times 1 \times 1 = 1$ $\quad 2^3 = 2 \times 2 \times 2 = 8$ $\quad 3^3 = 3 \times 3 \times 3 = 27$

$4^3 = 4 \times 4 \times 4 = 64$ $\quad 5^3 = 5 \times 5 \times 5 = 125$ $\quad 6^3 = 6 \times 6 \times 6 = 216$

The numbers 1, 8, 27, 64, 125, 216 are known as cube numbers of 1 , 2 , 3 , 4 , 5 , 6 respectively.

These cube numbers 1 , 8 , 27 , 64 , 125 , 216.... are also called **Perfect cubes.**

A number is a **Perfect cube** if the number can be expressed as the product of three same numbers.

Is 1000 a perfect cube?

Yes, because 1000 can be expressed as the product of three same numbers as $10\times10\times10$.

Cube roots :

If any number is expressed as $x \times x \times x$ then x is called the cube root of that number.

We know that $4^3 = 4 \times 4 \times 4 = 64$. Here 64 is called the cube of 4 and 4 is called the cube root of 64.

Similarly $2^3 = 2 \times 2 \times 2 = 8$

Here 8 is called the cube of 2 and 2 is called the cube root of 8.

Note: We find for every three same factors the cube root contains one factor.

The symbol $\sqrt[3]{\ }$ is used to denote the cube root of a number.

Thus $\sqrt[3]{64} = \sqrt[3]{4\times 4 \times 4} = 4$, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$

Cube root of x is also written as $x^{1/3}$ that is $x^{1/3} = \sqrt[3]{x}$

Example 45 :

Find the cube roots of (a) 125 (b) 512 (c) $\frac{1}{27}$

Solution :

We know that for every three same factors the cube root contains one factor.

(a) $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = \sqrt[3]{5^3} = 5$

(b) $\sqrt[3]{512} = \sqrt[3]{8 \times 8 \times 8} = \sqrt[3]{8^3} = 8$

(c) $\sqrt[3]{\frac{1}{27}} = \sqrt[3]{\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}} = \sqrt[3]{\left(\frac{1}{3}\right)^3} = \frac{1}{3}$

Example 46 :

Find $\sqrt[3]{10^2 \times 10^4}$

Solution :

$$10^2 \times 10^4 = (10 \times 10) \times (10 \times 10 \times 10 \times 10)$$
$$= \underbrace{10 \times 10 \times 10} \times \underbrace{10 \times 10 \times 10}$$
$$= 10^3 \times 10^3$$
$$\therefore \sqrt[3]{10^2 \times 10^4} = \sqrt[3]{10^3 \times 10^3}$$
$$= 10 \times 10$$
$$= 100$$

Example 47 :

Find the cube root of 1728

Solution :

First factorise 1728 into prime factors

$$\therefore 1728 = (2\times2\times2)\times(2\times2\times2)\times(3\times3\times3)$$

$$\sqrt[3]{1728} = \sqrt[3]{2^3 \times 2^3 \times 3^3}$$

$$= 2\times 2\times 3$$

$$= 12$$

$$\therefore \sqrt[3]{1728} = 12$$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Example 48 :

Find $\sqrt[3]{3375}$

Solution :

Factorise 3375 into prime factors.

$$\therefore 3375 = (5\times5\times5)\times(3\times3\times3)$$

$$\therefore \sqrt[3]{3375} = \sqrt[3]{5^3 \times 3^3}$$

$$= 5\times 3$$

$$= 15$$

$$\therefore \sqrt[3]{3375} = 15$$

5	3375
5	675
5	135
3	27
3	9
3	3
	1

Note: Using the idea of cubes and cube roots we can find the volume of the cube and from the volume we can find the length of the side of the cube.

Discuss: How many faces are there in a cube? What is the area of each face?

Exercise 1.7

1. ***Find the square of each of the following:***

   (a) 13 ***(b) 18*** ***(c) 22*** ***(d) -123***

2. ***Find the square root of***

   (a) 289 ***(b) 361*** ***(c) 6400*** ***(d) 3136***

3. ***Find the values of the following***

   (a) $\sqrt{12\times12}$ ***(b)*** $\sqrt{19\times19}$ ***(c)*** $\sqrt{10000}$ ***(d)*** $\sqrt{1}$

4. ***Find the value of***

   (a) $\sqrt{6^2\times8^2}$ ***(b)*** $\sqrt{2^2\times3^2\times4^2}$ ***(c)*** $\sqrt{7\times7^3}$ ***(d)*** $\sqrt{2^3\times2^5}$

5. *Find the cube of each of the following*

(a) 4	*(b) 6*	*(c) 12*	*(d) 37*

6. *Find the cube root of the following*

(a) $5 \times 5 \times 5$	*(b) $7 \times 7 \times 7$*	*(c) $4^3 \times 8^3$*	*(d) $5^2 \times 5^4$*

7. *Find the cube root of*

(a) 343	*(b) 729*	*(c) 1000*	*(d) 4096*
(e) 2744	*(f) 5832*		

THINGS TO REMEMBER

1. **A number is a perfect square if the number can be expressed as the product of two same numbers.**
2. **If any number is expressed as $x \times x$ then x is called the square root of that number.**
3. **In general, we can find two square roots for every number. They are positive square root and negative square root.**
4. **A number is a perfect cube if the number can be expressed as the product of three same numbers.**
5. **If any number is expressed as $x \times x \times x$ then x is called the cube root of that number.**

1.4 Approximations:

1.4.1 Rounding off the numbers
1.4.2 Significant digits

1.4.1 Rounding off the numbers :

On many occasions in our daily life, the approximate measures are needed. The approximate measure means the value which is not exactly given.

Now observe the following situations:

A person buys a television set for Rs. 7990. When he wants to convey this amount to others, he simply says that he has bought the television for Rs. 8000.

This is the approximate value which is given in thousands only.

Another person buys a pair of shoes for Rs. 499.95. This amount may be considered approximately as Rs. 500 for convenience.

A picture has the dimensions of 25.23 cm. long and 15.23 cm. wide. If we want to check the measurements with our ordinary scale, we cannot measure accurately as we have seen that our ordinary scale is marked in tenths of centimetre only. In such cases, we can check the length of the picture as 25.2 cm to the **nearest tenth** or 25 cm to the **nearest integer** value.

In the above situations we have taken the approximate values for our convenience. This type of taking the nearest value is called **'rounding off'** the digits.

Thus the approximate value corrected to the required number of digits is known as ' rounding off ' the digits.

While rounding off the numbers we adopt the following principles

(a) If the number next to the desired place of correction is less than 5, give the answer upto the desired place as it is.

(b) If the number next to the desired place of correction is 5 and greater than 5 add 1 to the number in the desired place of correction and give the answer.

Note: The symbol for approximation is usually denoted by $\doteq$. The equality symbol ' = ' is replaced by $\doteq$ when we use approximate values.

Example 49 :

Round off the following numbers to the nearest integer :

(a) 637.27 (b) 5998.36 (c) 250.05

Solution :

The above numbers are rounded off to the nearest integer as follows:

(a) $637.\underline{2}7 \doteq 637$ (b) $5998.\underline{3}6 \doteq 5998$

(c) $250.\underline{0}5 \doteq 250$

[Here, the tenth place in the numbers are less than 5. Therefore all the integers are left as they are.]

Example 50 :

Round off the following numbers to the nearest integer

(a) 535.73 (b) 2856.892 (c) 1999.95

Solution :

The above numbers are rounded off to the nearest integer as follows:

(a) $535.\underline{7}3 \doteq 536$ (b) $2856.\underline{8}92 \doteq 2857$

(c) $1999.\underline{9}5 \doteq 2000$

[Here, the tenth place in the numbers are greater than 5. Therefore the integer values are increased by 1 in each case.]

Example 51 :

Round off the following numbers to the first decimal place:

(a) 32.562 (b) 69.356 (c) 3464.2952 (d) 845.715

Solution :

The above numbers are corrected to the first decimal place (or corrected to tenth place value) as follows:

(a) $32.5\underline{6}2 \doteq 32.6$ (b) $69.3\underline{5}6 \doteq 69.4$

(c) $3464.2\underline{9}52 \doteq 3464.3$ (d) $845.7\underline{1}5 \doteq 845.7$

Example 52 :

Round off the following numbers to two decimal places:

(a) 14.7281 (b) 2.60215 (c) 35.6555 (d) 173.293

Solution :

The above numbers are rounded off to two decimal places as follows

(a) 14.72$\underline{8}$1 $\triangleq$ 14.73 (b) 2.60$\underline{2}$15 $\triangleq$ 2.60

(c) 35.65$\underline{5}$5 $\triangleq$ 35.66 (d) 173.29$\underline{3}$ $\triangleq$ 173 .29

Example 53 :

Round off the following numbers to three decimal places

(a) 423.85683 (b) 15.37418 (c) 7632.54085

Solution :

The above numbers are rounded off to three decimal places as follows

(a) 423.856$\underline{8}$3 $\triangleq$ 423 . 857 (b) 15.374$\underline{1}$8 $\triangleq$ 15.374

(c) 7632.5408$\underline{5}$ $\triangleq$ 7632 . 541

Discuss : Sometimes it is possible only to give approximate values.

(1) Can you give the exact population of our country?

(2) We never say that the distance between Chennai and Madurai is 496.13 kilometres. We represent this only by 496 kms. Why?

1.4.2 Significant digits :

Numbers are rounded off for various reasons because of inaccurate instruments, for ease of reference and to make them easier to remember. The most significant digits are always those whose **place values** are the greatest, that is, those on the **left.**

For example in the number 23.456 and 0.023456 ,

the number ' 2' is more significant than the number ' 3 ' and

the number '3 ' is more significant than the number ' 4 '.

The number of significant digits or 'figures' we choose, depends upon the place value of the number.

Suppose the price of a car is Rs. 3,24,250 when the number is rounded off to 1 significant figure we give it as Rs 3,00,000. If we want the same number rounded off to two significant figures, we say that the price is Rs 3,20,000. Similarly for 3 significant figures that amount is represented as Rs 3,24,000.

In this manner we can round off any number according to the number of significant figures as per the requirement.

Example 54 :

Write down the following numbers rounded off to

(i) 1 significant figure

(ii) 2 significant figures.

(a) 213 (b) 397 (c) 5088 (d) 1134 (e) 636.85

Solution :

(i) Using the principle of approximation, we round off the following numbers to 1 significant figure.

(a) $\underline{2}$13 $\triangleq$ 200 [Here we consider the first digit which has the highest place value]

(b) $\underline{3}97 \simeq 400$ (c) $\underline{5}088 \simeq 5000$

(d) $\underline{1}134 \simeq 1000$ (e) $\underline{6}36.85 \simeq 600.00$

(ii) Using the principle of approximation, we round off the following numbers to 2 significant figures.

(a) $\underline{21}3 \simeq 210$ [Here we consider the first two digits, which have the highest place values]

(b) $\underline{39}7 \simeq 400$ (c) $\underline{50}88 \simeq 5100$

(d) $\underline{11}34 \simeq 1100$ (e) $\underline{63}6.85 \simeq 640.00$

Example 55 :

To how many significant figures have the following measurements be given?

(a) 5 cm (b) 7.2cm (c) 4.00m (d) 496 km

(e) 23.46 m (f) 2000

Solution :

(a) 5 cm	has	1 significant figure
(b) 7.2 cm	has	2 significant figures
(c) 4.00 m	has	1 significant figure
(d) 496 km	has	3 significant figures
(e) 23.46m	has	4 significant figures
(f) 2000	has	1 significant figure

Exercise 1.8

1. ***Round off the following numbers to the nearest integer***
   (a) 42.376 (b) 985.63 (c) 502.08 (d) 1058.95
   (e) 547.25 (f) 199.87
2. ***Round off the following numbers to one decimal place***
   (a) 8.37 (b) 12.835 (c) 23.962 (d) 5.2004
3. ***Round off the following numbers to two decimal places***
   (a) 41.398 (b) 512.026 (c) 16.5555 (d) 4.3520
4. ***Round off the following numbers to three decimal places***
   (a) 2.1539 (b) 57.8845 (c) 27.3002 (d) 29.9997
5. ***Write down the following numbers rounded off to 1 significant figure and 2 significant figures***
   (a) 342.754 (b) 2664.05 (c) 12.576 (d) 5486
   (e) 1999.95
6. ***To how many significant figures have the following measurements been given?***
   (a) 15 cm (b) 8.20m (c) Rs 6.00 (d) 356 km
   (e) 7000 m
7. ***To how many significant digits are the following figures stated?***
   (a) The height of Mount Everest is 8882 metres.
   (b) The circumference of the earth is 40,000 kms.

THINGS TO REMEMBER

1. **The approximate value corrected to the required number of digits is known as ' rounding off ' the digits.**
2. **While rounding off the numbers we adopt the following rules:**
   (a) If the number next to the desired place of correction is less than 5, give the answer upto the desired place as it is.
   (b) If the number next to the desired place of correction is 5 and greater than 5, add 1 to the number in the desired place of correction and give the answer.
3. **The most significant digits are always those whose place value is the greatest, that is those on the left.**

UNIT TEST –1

Part – I *(10 × 1 =10)*

Choose the correct answer :

1. *The additive inverse of –7 is*

   (a) 0 *(b) – 7* *(c) + 7* *(d) 1*

2. *(–1) (10 – 6) is*

   (a) – 10 *(b) + 6* *(c) + 4* *(d) – 4*

3. $\left(-\frac{1}{4}\right) + \left(-\frac{3}{4}\right)$ *is equal to*

   (a) 1 *(b) – 1* *(c)* $-\frac{1}{4}$ *(d)* $-\frac{3}{16}$

4. *The cube root of 64 is*

   (a) 4 *(b) 8* *(c) 16* *(d) 32*

5. *The number 24.647 rounded off to two decimal place is*

   (a) 24.64 *(b) 24.65* *(c) 25.00* *(d) 25.01*

Fill in the blanks:

6. *15 – (13 – 8) =* _____

7. *The greater among* $-\frac{4}{5}$ *and* $-\frac{2}{5}$ *is* _____

8. *The multiplicative inverse of* $-\frac{5}{4}$ *is* _____

9. *The square root of* 3×3^3 *is* _____

10. *The number of significant figures stated in the number 1,86,000 is* _____

Part – II *(10 × 2 = 20)*

Answer the following in one or two steps:

11. Find all the integers between – 5 and – 1

12. Divide (– 407) by 11.

13. Simplify (16 – 26) ÷ (3 – 13)

14. Which is greater ? $\frac{7}{8}$ *or* $\frac{7}{12}$

15. Express the rational number $\frac{25}{-85}$ *in standard form.*

16. Fill in the boxes:

$$\frac{-4}{8} = \frac{+8}{\square} = \frac{\square}{+24}$$

17. Subtract : $\frac{5}{32} - \frac{7}{48}$

18. Divide: $7\frac{1}{5} \div \left(-3\frac{3}{5}\right)$

19. Find the cube root of $2^4 \times 2^5$

20. Write down the number 735. 25 to 1 significant figure and 2 significant figures.

Part – III

(5 × 4 = 20)

Give detailed answers :

21. Simplify : $\frac{72 \times (-15) \times (-4)}{(-5) \times (2)^3}$

22. Simplify: $\left(-4\frac{2}{13}\right) + \left(1\frac{3}{26}\right) + \left(2\frac{1}{2}\right)$

23. Simplify: $\left(2\frac{5}{6}\right) \times \left(-4\frac{1}{2}\right) \times \left(1\frac{1}{3}\right)$

24. The product of two rational numbers is $-\frac{15}{22}$*. If one of the numbers is* $\frac{3}{22}$*, find the other number.*

25. Find the cube root of 15625.

2. EVERYDAY ARITHMETIC

The part of mathematics that deals with numbers and counting or calculation is known as Arithmetic. For a person belonging to any profession, knowledge of Arithmetic is essential to cope up with the needs of daily life.

A good command of Arithmetic is essential for the skilled mechanic, for the up-to-date farmer, for the progressive professional man, for the efficient house wife and for the brilliant students.

Therefore Arithmetic has a greater value in day-to-day life.

2.1 Variation
2.2 Percentages
2.3 Finance

2.1 Variation :

2.1.1 Direct variation – Revision
2.1.2 Inverse variation

In the previous standard we have studied the concept of 'Direct Variation'. Now in this chapter we will study the concept of 'Inverse Variation' and its applications.

2.1.1 Direct variation – Revision :

If two quantities vary uniformly such that when one increases the other also increases or when one decreases the other also decreases then they are said to be in direct variation.

Example 1 :

The cost of a book is Rs.10. Find the cost of 2 books, 5 books and 10 books.

Solution :

Number of books	1	2	5	10
Cost of books (Rs.)	10	20	50	100

From the above example we observe that $\frac{1}{10} = \frac{2}{20} = \frac{5}{50} = \frac{10}{100}$

$$\frac{\text{Number of books}}{\text{Cost of books}} = \frac{1}{10}$$

$$\therefore \text{Number of books} = \frac{1}{10} \times \text{Cost of books}$$

Further we observe that if the number of books increases the cost also increases. If the number of books decreases then the cost also decreases. In both the cases the relation is in direct variation.

Example 2 :

The cost of 5 fruits is Rs.25. Find the cost of 15 fruits.

Solution :

Number of fruits	Cost of fruits
5 ↓	25 ↓
15 ↓	x ↓

Here $5 : 15 = 25 : x$

$$5 \times x = 15 \times 25$$

$$x = \frac{15 \times 25}{5} = 75$$

$\therefore$ The cost of 15 fruits = Rs. 75

2.1.2 Inverse variation :

Consider the following :

1. A person travels from one place to another place in 5 hours at a speed of 40 km per hour. Another person reaches the same place in 4 hours at a speed of 50 km per hour. We observe that as the speed increases, time decreases.
2. 10 men can do the work in 30 days and 15 men can do the same work in 20 days. We observe that as the number of men increases the number of days decreases.

From the above two examples we observe that when one quantity increases the other quantity decreases uniformly. Here the two quantities are said to be in inverse variation.

Example 3 : (Time and work)

Six workers can do a job in 12 days. In how many days eight workers will do the same job ?

Solution :

Number of workers	Number of days
6 ↓	12 ↑
8 ↓	x ↑

We know that as the number of workers increases the number of days decreases.

$\therefore$ The two quantities are in inverse variation.

$$\text{i.e } 6 : 8 = x : 12$$

$$8 \times x = 6 \times 12$$

$$x = \frac{6 \times 12}{8} = 9$$

$\therefore$ 8 workers can do the same job in 9 days.

Example 4 : (Time and Speed)

A person travels from Chennai to Madurai in 9 hours at a speed of 50 km / hour. What is the time taken to reach Madurai from Chennai at a speed of 75 km / hour?

Solution :

Speed (km)	Time (Hrs.)
50 ↓	9 ↑
75 ↓	x ↑

As speed increases time decreases.

$\therefore$ It is in inverse variation.

Here $50 : 75 = x : 9$

$$75 \times x = 9 \times 50$$

$$\therefore \quad x = \frac{9 \times 50}{75} = 6$$

$\therefore$ Time taken to reach Madurai from Chennai at a speed of 75 km / hr = 6 hrs.

Example 5 : (Article and Cost)

A person purchases 20 chocolates at Rs.5 each. With the same amount how many chocolates can be purchased at Rs.10 each ?

Solution :

Cost per chocolate	Number of chocolates
5 ↓	20 ↑
10 ↓	x ↑

As cost increases number of chocolates decreases.

$\therefore$ It is in inverse variation

Here $5 : 10 = x : 20$

$$10 \times x = 5 \times 20$$

$$x = \frac{5 \times 20}{10} = 10$$

$\therefore$ Number of chocolates for the same amount = 10

Exercise 2.1

1. ***15 men can do a job in 5 days. How many men are needed to do the same job in 3 days ?***
2. ***30 workers can dig a well in 15 days. How many days are needed for 50 workers to dig the same well ?***
3. ***8 men can paint a house in 5 days. How many men will be required to paint the same house in 4 days ?***
4. ***Saravanan goes to his school by cycle at an average speed of 10 km / hr. It takes 12 minutes to reach the school. If he wants to reach the school in 15minutes, what should be his average speed ?***
5. ***Moving at the rate of 60 km / hr a car reaches a place in 2 hours and 20 minutes. How long will it take to complete this journey if the speed is increased to 70 km / hr ?***
6. ***A hostel has food grains for 50 students for 20 days . If 10 students leave the hostel, how long will the food grains last ?***

7. *In a camp of 160 students provisions are available for 10 days. If 40 more students join the camp, how long will the provisions last ?*
8. *A business man has just enough money to buy 50 articles worth Rs.12 each. If the cost of each article increases by Rs. 3 , then how many articles can be bought with the same amount of money ?*
9. *A certain quantity of sweets is packed into 50 packets each containing 200 gms. If the same quantity of sweets is to be packed into packets of 250 gms of each, how many packets do we get ?*
10. *A car travelling at 60 km / hr takes 7 hours to reach a place. If the car travels at 70 km / hr how much time will it take to reach the same place ?*

2.2 Percentages :

2.2.1	**Percentage – Revision**
2.2.2	**Commission**
2.2.3	**Discount**
2.2.4	**Sales tax**

2.2.1 Percentage – Revision :

We have studied in the previous standard that a percentage is a fraction whose denominator is 100. The symbol for percentage is %. Also we know how to convert a percentage into a fraction or decimal and also to find the percentage of a given number.

Exercise 2.2

1. Convert the following into percentages.

(a) $\frac{3}{4}$ *(b)* $\frac{1}{2}$ *(c)* $\frac{2}{5}$ *(d)* $3\frac{1}{5}$ *(e)* $2\frac{1}{2}$

(f) $4\frac{1}{4}$ *(g) 0. 8* *(h) 0.25* *(i) 1.50*

2. Convert the following into fractions.

(a) 60 % *(b) 35 %* *(c) 80 %*

3. Find the following:

(a) 20 % of 200 *(b)* $13\frac{1}{3}$ *% of 300* *(c) 7.5 % of 50*

4. Vivek saves 10 % of his salary. What percentage of his salary does he spend ?

5. Find the salary of a man , if 25 % of his salary is Rs. 1250 ?

2.2.2 Commission :

When we want to buy or sell a house or a land or a vehicle , some amount is given to the person who helps in the business. This amount is known as commission or brokerage. The person who receives the commission is known as a broker. The broker receives the commission both from the buyer and from the seller.

Commission is always given in percentage. For example, the sales representatives of various companies such as medicines, soaps, machines etc. are given commission on the amount of sales.

Example 6 :

Deepak sells a house through a broker for Rs.5,00,000 paying a brokerage of 2 %. Find the amount of brokerage.

Solution :

The selling price of the house = Rs.500000

Rate of brokerage = 2 %

$$\text{Amount of brokerage} = \frac{2}{100} \times 500000$$

$$= 10000$$

$\therefore$ The amount of brokerage = Rs.10,000

Example 7 :

A broker sold a motor bike for Rs. 12,000 and received a commission of Rs. 600. Find the rate of commission.

Solution:

The selling price of the motor bike = Rs.12000

Amount of commission = Rs.600

For Rs. 12000 the commission = Rs.600

$$\text{For Rs. 100 the commission} = \frac{600}{12000} \times 100 = 5$$

$\therefore$ Rate of commission = 5 %

Example 8 :

A broker helps to sell a house and gets a commission of Rs. 2,500. If the rate of commission is 2 %, find the selling price of the house.

Solution :

Amount of commission = Rs. 2500

Rate of commission = 2 %

If Rs. 2 is the commission , the selling price = Rs. 100

$$\text{If Rs. 2500 is the commission , the selling price} = \frac{100}{2} \times 2500$$

$$= 125000$$

$\therefore$ Selling price of the house = Rs. 1,25,000

Exercise 2.3

1. ***Seema sold her land through a broker for Rs. 50,000 paying a brokerage of 2 %. Find the amount of brokerage?***
2. ***A medical company gives to its employee a monthly salary of Rs. 1,000 and in addition an incentive commission of 15 % on the amount of sales. In a month the amount of sales by a***

representative is Rs. 10,000 worth of medicines. How much will he get as total salary in that month?

3. *An agent has helped to sell a shop worth Rs. 2,00,000 at a commission of 3 % from the seller and 1 % from the buyer. Find the total amount of commission he gets.*
4. *A broker sold a car for Rs. 75,000 and received a commission of Rs. 3,000. Find the rate of commission.*
5. *An agent received Rs. 2,500 as commission by selling a house worth Rs. 1,00,000. Find the rate of commission.*
6. *A broker sold a car for Rs. 60,000 and received a commission of Rs. 1,200. Find the rate of commission.*
7. *A broker helps to sell a land and gets a commission of Rs. 3,000. If the rate of commission is 2 % find the selling price of the land.*
8. *An agent receives an amount of Rs. 5,000 as commission by selling a car. If the rate of commission is 5 % find the selling price of the car.*
9. *Murugan bought a house for Rs. 2,25,000. He paid 1 % commission over the cost price. How much did he spend for the house on the whole?*
10. *Fill in the blanks :*

	Name of the article	*Selling price*	*Rate of commission*	*Amount of commission*
(a)	*Washing machine*	*Rs. 8,000*	*2 %*	_____
(b)	*Television*	*Rs. 15,000*	_____	*Rs. 450*
(c)	*Refrigerator*	_____	$1\frac{1}{2}$ %	*Rs. 150*

2.2.3 Discount :

During the festivals like Pongal, Deepavali, Christmas and Ramzan, we have seen that rebates of 10 % or 20 % or 30 % are offered by Co-optex, Khadi and other shops for various items. Also in the Tamil month "Aadi" we have seen 10 % to 50 % discount offered by merchants for various items in order to increase their sales and to introduce the new products.

The amount reduced from the marked price of the item or the rebate given is called discount.

The list price or the catalogued price is known as marked price.

Discount is always calculated on marked price.

1. Selling price = Marked price – Discount
2. Discount = Marked price – Selling price.

Example 9 :

The marked price of a television is Rs. 12,000 and the shopkeeper allows a discount of 5 % on it. Find the selling price.

Solution :

Marked price of the television = Rs. 12000

Rate of discount = 5 %

$$\text{Amount of discount} = \frac{5}{100} \times 12000$$

$$= \text{Rs. } 600$$

$\therefore$ Selling price of the television = Marked price – Discount

= 12000 – 600

= 11400

$\therefore$ Selling price of the television = Rs. 11,400

Example 10 :

A bicycle marked at Rs. 1,500 is sold for Rs. 1,350. What is the percentage of discount ?

Solution :

Marked price = Rs. 1500

Selling price = Rs. 1350

Amount of discount = Marked price – selling price

= 1500 – 1350

= 150

Discount for Rs. 1500 = Rs. 150

$$\text{Discount for Rs. 100} = \frac{150}{1500} \times 100 = 10$$

$\therefore$ Rate of discount = 10 %

Example 11 :

A shopkeeper sells a fan at 6 % discount. Find the marked price of the fan whose selling price is Rs. 846.

Solution :

Since the discount is given in percentage the marked price is taken as Rs. 100

Let the marked price = Rs. 100

Rate of discount = 6 %

$$\text{Amount of discount} = \frac{6}{100} \times 100 = \text{Rs. } 6$$

Selling price = Marked price – discount

= 100 – 6

= Rs. 94

If the selling price is Rs. 94 , then the marked price = Rs. 100

If the selling price is Rs. 846 , then the marked price $= \frac{100}{94} \times 846$

= 900

$\therefore$ The marked price of the fan = Rs. 900

Exercise 2.4

1. ***The marked price of a watch is Rs. 950. If the rate of discount is 10 % , find the selling price of the watch.***
2. ***The price of a book is Rs. 150. The seller allows a discount of 15 % on it. Find the selling price of the book***

3. *The catalogued price of a colour television is Rs. 14,500. If the dealer allows a discount of 5 % , what is the selling price of the colour television ?*
4. *A watch marked at Rs. 1,000 is sold for Rs. 900. What is the percentage of discount ?*
5. *The list price of a computer is Rs. 40,000 and it is sold for Rs. 39,000. What is the rate of discount ?*
6. *Find the rate of discount being given on a chair whose marked price is Rs. 1,200 and selling price is Rs. 1,000*
7. *A cycle is sold for Rs. 1,000 after allowing a discount of Rs. 200. What is the percentage of discount ?*
8. *A shirt is sold for Rs. 135 at 10 % discount. What is the marked price of the shirt ?*
9. *A shopkeeper allows 20 % discount on his goods. Find the marked price of an article which was sold by him for Rs. 800.*
10. *Fill in the blanks :*

	Name of the article	*Marked price*	*Selling price*	*Discount*
(a)	*Book*	*Rs. 80*	*Rs. 68*	*Rs. ____*
(b)	*Table*	*Rs. 500*	*____*	*10 %*
(c)	*Television*	*____*	*Rs. 11,400*	*5 %*

2.2.4 Sales tax :

Look at the bill given below :

CASH BILL

SAMY ELECTRICALS

15, MAIN BAZAAR

BODINAYAKANUR.

No : 6666 DATE : 27. 09. 03

S. No.	Item	Rate		Quantity	Amount	
		Rs.	P		Rs.	P
1.	Fan	1,100	00	1	1,100	00
2.	Radio	500	00	1	500	00
				Total	1,600	00
				Sales tax @ 12%	192	00
				Net total	1,792	00

xxxxxx

Signature

The sales price of the two items is Rs. 1,600.

In the bill, below the total amount there is an entry as sales tax @ 12 %

What is sales tax ?

Sales tax is the tax levied on the sale made by a seller at the time of selling the product. When the buyer buys the commodity the sales tax is paid by him together with the price of the commodity.

It is normally levied at a fixed percentage on the selling price. However this percentage is different for different commodities.

The prevailing rate of sales tax for some commodities in the year 2003 is as follows:

1). Electrical instruments – 12 %

2). Domestic appliances – 12 %

3). Petrol – 30 %

4). Diesel – 25 %

5). Air conditioner – 20 %

6). Chemicals – 12 %

Meanwhile some commodities like rice, sugar, milk, common salt, pen, pencils, books are exempted from sales tax.

Calculation of sales tax :

$$\text{Sales tax} = \frac{\text{Rate of sales tax}}{100} \times \text{Amount of sales}$$

Total amount of the bill = Total amount of sales + Sales tax

$$\text{Rate of sales tax} = \frac{\text{Sales tax}}{\text{Total amount of sales}} \times 100$$

Example 12 :

A dealer purchased electrical instruments for Rs. 6,000. If the rate of sales tax is 12 % find the amount to be paid by the dealer.

Solution :

Amount of electrical instruments = Rs. 6000

Rate of sales tax = 12 %

$$\therefore \text{Sales tax} = \frac{12}{100} \times 6000$$

$$= \text{Rs. } 720$$

Total amount to be paid = Amount of sales + Sales tax

$$= 6000 + 720$$

$$= 6720$$

$\therefore$ The total amount to be paid by the dealer = Rs. 6,720

Example 13 :

The sale proceeds of a commodity is Rs. 2,000. If the buyer paid a sum of Rs. 2,200 for that commodity find the rate of sales tax.

Solution :

Sale proceeds = Rs. 2000

Total amount paid = Rs. 2200

$\therefore$ Sales tax = Total amount paid − Sale proceeds

= 2200 − 2000

= Rs. 200

$$\text{Rate of sales tax} = \frac{\text{Sales tax}}{\text{Total amount of sales}} \times 100$$

$$= \frac{200}{2000} \times 100 = 10$$

$\therefore$ Rate of sales tax = 10 %

Exercise 2.5

1. *A dealer purchased chemicals for Rs. 5,000 . If the rate of sales tax is 12 % find the total amount paid by the dealer.*
2. *Badri purchased an air conditioner for Rs. 20,000. If the rate of sales tax is 20 % find the total amount paid by him.*
3. *A dealer purchased 1,000 litres of petrol at Rs. 30.50 per litre and 1,500 litres of diesel at Rs. 21 per litre. If the rate of sales tax for petrol is 30 % and for diesel is 25 % find the total sales tax.*
4. *Ramesh purchased domestic appliances for Rs. 12,000 and electrical goods for Rs. 15,000. If the rate of sales tax for the domestic appliances is 12 % and that of electrical goods is 10 % find the total amount paid by him.*
5. *Aparna ordered a commodity for Rs. 20,000 and paid a sum of Rs. 26,000 including the sales tax. Find the rate of sales tax.*
6. *An agent bought a commodity for Rs. 15,000. If he paid a sum of Rs. 16,500 including sales tax find the rate of sales tax.*

THINGS TO REMEMBER

1. **Commission is always given in percentage.**
2. **Discount is always calculated on marked price.**
3. **Discount = Marked price − Selling price.**
4. **Selling price = Marked price − Discount.**
5. **Sales tax** $= \frac{\text{Rate of sales tax}}{100} \times$ **Amount of sales.**
6. **Total amount of the bill = Total amount of sales + Sales tax.**
7. **Rate of Sales tax** $= \frac{\text{Sales tax}}{\text{Total amount of sales}} \times 100.$

2.3 Finance :

2.3.1 Simple Interest
2.3.2 Recurring Deposits

2.3.1 Simple Interest :

In the previous standard we have studied about simple interest using unitary method. Now we study the method of calculating the simple interest using the formula.

Simple interest (I) $= \frac{P\,n\,r}{100}$ where P = Principal

n = Number of years.

r = Rate of interest per annum.

(a) Amount = Principal + Interest

$A = P + I$

(b) Interest = Amount – Principal

$I = A - P$

(c) Number of years , $n = \frac{100 \times I}{P \times r}$

(d) Rate of interest per annum, $r = \frac{100 \times I}{P \times n}$

(e) Principal , $P = \frac{100 \times I}{n \times r}$

Example 14 :

Madan invested Rs. 5,000 for 3 years at the rate of 6 % per annum. Find the simple interest and the amount received by him at the end of 3 years.

Solution:

Here Principal , P = Rs.5000
Number of years , n = 3
Rate of interest , r = 6 %

Simple interest, $I = \frac{P\,n\,r}{100}$

$= 5000 \times 3 \times \frac{6}{100}$

= Rs. 900

Amount, A = Principal + Interest
= 5000 + 900
= Rs. 5900

∴ The simple interest = Rs .900
Amount = Rs.5,900

Example 15 :

Sathya invested Rs. 6,000 for 5 years. He received the simple interest of Rs. 1,500. Find the rate of interest.

Solution :

Here Principal, P = Rs.6000

Number of years , n = 5

Simple interest , I = Rs.1500

Rate of interest , r = ?

We know that rate of interest, $r = \frac{100 \times I}{P \times n}$

$= \frac{100 \times 1500}{6000 \times 5} = 5$

$\therefore$ Rate of interest = 5 %

Example 16 :

Vimala invested Rs. 10,000 at the rate of 5 % simple interest per annum. She received Rs.11,000 after some years. Find the number of years.

Solution :

Here Amount , A = Rs. 11000

Principal, P = Rs. 10000

Rate of interest , r = 5 %

Number of years , n = ?

We know that amount , $A = P + I$

$\therefore I = A - P$

$= 11000 - 10000$

= Rs. 1000

We know that number of years , $n = \frac{100 \times I}{P \times r}$

$= \frac{100 \times 1000}{10000 \times 5} = 2$

$\therefore$ Number of years = 2

Example 17 :

Anitha deposited a certain amount of money for 5 years at the rate of 5 % per annum. If she received Rs. 5,000 as interest find the money deposited.

Solution:

Here Number of years, n = 5

Rate of interest , r = 5 %

Simple interest , I = Rs. 5000

Principal , P = ?

We know that Principal, $P = \frac{100 \times I}{n \times r}$

$= \frac{100 \times 5000}{5 \times 5}$

$= 20000$

$\therefore$ Money deposited = Rs. 20,000

Exercise 2.6

1. *Raj deposited Rs. 15,000 for 5 years at the rate of 6 % per annum. Find the interest received by him.*
2. *Karthik deposited Rs. 20,000 for $3\frac{1}{2}$ years at the rate of 5 % per annum. Find the total amount received by him.*
3. *Suresh invested Rs. 10,000 for $4\frac{1}{2}$ years at the rate of 7 % per annum. Find the total amount received by him.*
4. *Deepak invested Rs.12,000 for 3 years and received Rs. 2,160 as interest. Find the rate of interest.*
5. *Vivek invested Rs. 25,000 for 5 years and received Rs. 10,000 as interest. Find the rate of interest.*
6. *Tamilarasi deposited Rs.20,000 for 6 years and received Rs. 26,000 as the amount after 6 years. Find the rate of interest.*
7. *In how many years will Rs. 16,000 earn an interest of Rs.1,600 at the rate of 5 % per annum ?*
8. *In how many years will Rs. 50,000 yield an interest of Rs 12,000 at the rate of 6 % per annum ?*
9. *In how many years will a sum of Rs. 5,000 amount to Rs. 5,800 at the rate of 8 % per annum ?*
10. *Find the principal which earns Rs. 250 as simple interest in $2\frac{1}{2}$ years at 10 % per annum ?*
11. *If the interest for 2 years is Rs. 1,200 at 12 % per annum, find the principal.*
12. *A sum of money doubles in 10 years. Find the rate of interest.*
13. *Fill in the blanks :*

	Principal (P) Rs.	Number of years (n)	Rate of interest (r)	Interest (I) Rs.	Amount (A) Rs.
(a)	2,500	2	5 %	_____	_____
(b)	4,000	_____	6 %	720	_____
(c)	7,500	$2\frac{1}{2}$	_____	750	_____
(d)	_____	4	10 %	2,000	_____
(e)	_____	_____	8 %	1,440	13,440

2.3.2 Recurring Deposits :

If a person wants to save for his children's education or marriage or wants to buy a car or a house, he should deposit a small amount of money in the bank or in the post office every month for a certain period.

The bank or post office repays the total amount deposited together with the interest for the amount deposited at the end of the period. This type of deposit is known as **Recurring Deposit.**

The interest on recurring deposit is calculated using simple interest method. We can pay the monthly deposit on any day of the month.

A person deposits Rs. 500 every month for a period of 60 months in a bank.

The interest is calculated for 60 months for the first month's deposit.

The interest is calculated for 59 months for the second month's deposit.

The interest is calculated for 58 months for the third month's deposit .

.....

.....

The interest is calculated for one month for the last month's deposit.

Even though the same amount is deposited every month, interest is calculated for different periods.

To find the formula for calculating the recurring deposit interest :

P – Principal - (monthly instalment)

r – Rate of interest per annum

n – Number of months

Rs. P deposited in the first month will earn the interest for 'n' months .

Rs. P deposited in the second month will earn the interest for '(n –1)' months .

Rs. P deposited in the third month will earn the interest for '(n –2)' months .

.....

.....

Rs. P deposited in the $(n-1)^{th}$ month will earn the interest for 2 months

Rs. P deposited in the n^{th} month will earn the interest for 1 month.

$\therefore$ On the whole Rs. P will earn the interest for [n + (n–1) + (n–2) +..... + 2 + 1] months.

$$= \frac{n\,(n+1)}{2} \text{ months} \qquad \left[\text{since } 1+2+.....+n = \frac{n\,(n+1)}{2}\right]$$

$$= \frac{n\,(n+1)}{2 \times 12} \text{ years}$$

$\therefore$ Simple interest for $\frac{n\,(n+1)}{2 \times 12}$ years at the rate of r % per annum for the principal P is

$$I = P \times \frac{n\,(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= \frac{Pn\,(n+1)\,r}{2 \times 12 \times 100}$$

Total amount, A = Principal × Number of instalments + Interest

$$A = P \times n + I$$

$$= Pn + I$$

Important Note :

In the calculation of interest for recurring deposits ' n ' is the number of months for which the monthly instalment is paid.

Activity : Try to start a recurring deposit by paying monthly instalments in the nearest bank or post office from your pocket money.

Example 18 :

Jothi deposits Rs. 100 at the beginning of every month for 6 years in a bank. Find the amount that she will get back at the end of 6 years if the bank gives 8 % interest per annum.

Solution :

Amount deposited every month , P = Rs. 100

Number of months , n = 72 (since 6 years = 6 × 12 = 72 months)

Rate of interest, r = 8 %

$$\text{Interest , I} = \frac{Pn\ (n+1)\ r}{2 \times 12 \times 100}$$

$$= \frac{100 \times 72 \times 73 \times 8}{2 \times 12 \times 100}$$

$$= \text{Rs. } 1{,}752$$

$$A = Pn + I$$

$$= (100 \times 72) + 1752$$

$$= 7200 + 1752 = 8952$$

∴ Amount received by Jothi at the end of 6 years = Rs. 8,952

Example 19 :

Kumaran deposited Rs. 200 at the beginning of every month for 5 years in a post office. If he received Rs. 15,050 at the end of 5 years find the rate of interest.

Solution :

Total Amount received , A = Rs. 15,050

Number of months, n = 60 (since 5 years = 60 months)

Amount deposited = P × n

$$= 200 \times 60$$

$$= 12000$$

Interest , I = A − (P × n)

$$= 15050 - 12000$$

$$= 3050$$

$$I = \frac{Pn\ (n+1)\ r}{2 \times 12 \times 100}$$

$$3050 = \frac{200 \times 60 \times 61 \times r}{2 \times 12 \times 100}$$

$$r = \frac{3050 \times 2 \times 12 \times 100}{200 \times 60 \times 61}$$

$$= 10$$

∴ Rate of interest = 10 %

Exercise 2.7

1. *Seema deposits Rs. 300 at the beginning of every month for 6 years in a bank. If the rate of interest is 10 % find the amount that she will receive at the end of 6 years.*
2. *Babu deposits Rs. 400 at the beginning of every month in a Post Office for 2 years. If the rate of interest is 12 % find the amount he will receive at the end of 2 years.*
3. *Meena deposits Rs. 500 at the beginning of every month for 5 years in a bank. If the rate of interest is 6 % find the interest she will receive at the end of 5 years.*
4. *Peter deposited Rs. 200 at the beginning of every month for 6 years in a post office. If he received Rs. 17,904 at the end of 6 years , find the rate of interest.*
5. *Syed deposited Rs. 600 at the beginning of every month in a bank for 5 years. At the end of 5 years he received Rs. 41,490. Find the rate of interest.*
6. *Viji deposited Rs. 700 at the beginning of every month in a bank for 6 years. If at the end of 6 years she received Rs. 64,197, calculate the rate of interest.*

THINGS TO REMEMBER

1. Simple interest $(I) = \frac{P\,n\,r}{100}$

   where P = Principal
   n = Number of years
   r = Rate of interest per annum.
2. Amount = Principal + Interest

   (i.e) A = P + I
3. Interest = Amount – Principal

   (i.e) I = A – P
4. Number of years , $n = \frac{100 \times I}{P \times r}$
5. Rate of interest per annum, $r = \frac{100 \times I}{P \times n}$
6. Principal, $P = \frac{100 \times I}{n \times r}$
7. Interest for recurring deposit , $I = \frac{P\,n\,(n+1)\,r}{2 \times 12 \times 100}$

   where P – Principal (monthly instalment)
   r – Rate of interest per annum
   n – Number of months

8. Amount received in recurring deposit , A = P n + I

$$\text{where } I = \frac{P\,n\,(n+1)\,r}{2 \times 12 \times 100}$$

UNIT TEST – 2

Part – I *(10 × 1 = 10)*

Choose the correct answer:

1. *4 persons complete a job in 2 days. The number of days required to complete the same job by 2 persons is*

   (a) 1 *(b) 2* *(c) 3* *(d) 4*

2. *5 % of Rs. 2,000 is*

   (a) Rs. 50 *(b) Rs. 100* *(c) Rs. 200* *(d) Rs. 500*

3. *The marked price of a radio is Rs. 750 and it is sold for Rs. 680. The discount allowed is*

   (a) Rs. 50 *(b) Rs. 80* *(c) Rs. 70* *(d) Rs. 100*

4. *The sales tax for Rs. 5,000 at the rate of 12 % is*

   (a) Rs. 500 *(b) Rs. 600* *(c) Rs. 700* *(d) Rs. 800*

5. *Simple interest for Rs. 5,000 at the rate of 5 % for 2 years is*

   (a) Rs. 250 *(b) Rs. 500* *(c) Rs. 750* *(d) Rs. 800*

Fill in the blanks :

6. *When the speed increases the time decreases. They are in _____ variation.*
7. *Selling price = Marked price -- _____*
8. *Rate of sales tax for rice is _____*
9. *Simple interest for Rs. 1,000 at the rate of 10 % for 3 years is _____*
10. *In recurring deposit, A = _____*

Part – II *(5 ×2 = 10)*

Answer in one or two steps :

11. *Eight persons complete a job in 6 days. In how many days can twelve persons complete the same job?*

12. *A broker sold a car for Rs. 50,000. If the rate of commission is 2 % , find the commission received by him.*

13. *A bicycle marked at Rs. 1,200 is sold for Rs. 1,100. What is the percentage of discount?*

14. *In how many years will Rs. 5,000 amount to Rs. 6,000 at the rate of 5 % per annum?*

15. *Prema deposits Rs. 100 every month for 5 years. If the rate of interest is 6 % per annum find the interest she receives at the end of 5 years.*

Part – III *(5 × 5 = 25)*

Give detailed answers :

16. *An agent sold a sewing machine for Rs. 5,000 and received a commission of 2 % from the seller and 1 % from the buyer. Find the total commission received by the agent.*

17. *An article is sold for Rs. 1,900 at 5 % discount. What is the marked price of the article?*

18. *Mani purchased domestic appliances for Rs. 10,000 and electrical goods for Rs. 5,000. If the rate of sales tax for the domestic appliances is 12 % and that of electrical goods is 10%, find the total amount paid by him.*

19. *Kavitha deposited Rs 10,000 for* $5\frac{1}{2}$ *years at the rate of 6 % per annum. Find the total amount received by her.*

20. *Selvi deposited Rs. 500 at the beginning of every month for 5 years in a bank. If she received Rs. 34,575 at the end of 5 years , find the rate of interest.*

3. MEASUREMENTS

Measurement plays not only a vital role in all walks of life but also it plays an essential role in human life. Measurement is a basis for comparison. To compare and compete, to detect and rectify we need a basic measurement in one unit or other. There are different units for different measurements. Nothing can be said about any quantity without the actual measurement in appropriate unit. We study about some important measurements in the following chapter.

3.1 Planning
3.2 Area and Perimeter
3.3 Volumes

3.1 Planning :

3.1.1 Necessity of Planning
3.1.2. Railway Time-table
3.1.3 Bus Time –table

3.1.1 Necessity of Planning :

Planning is needed for successful execution of any activity. A family has to plan its monthly expenditure to limit its expenses within the income. When we go on an excursion we have to plan the places to be visited, the mode of travelling (whether to travel by train or by bus) and the period and place of staying and so on. Only those who plan their journey reserve their tickets in advance and travel comfortably. In practical life families who plan their trips well in advance enjoy the journey where as families who do not plan may meet with inconveniences. In all walks of life people who plan their activities are successful and people who do not plan their activities are not successful.

"All that begins well, ends well"

3.1.2 Railway Time-table :

The passengers and tourists need to know certain vital (important) information in advance to plan their journey. For our convenience the Indian Railways has published a railway guide which contains a number of tables. These tables contain informations regarding train number and name, departure and arrival time, name of the intermediate stations between the boarding place and destination, the distance between stations, fares for different distances and so on.

Reading a table from Railway Time-table :

We all know how to read a school time-table. Can you read a railway time-table or a bus time-table? The time-tables worked out by the Railway or the Interstate Bus Service are planned according to the 24 hour clock time.

TABLE NO.71 – CHENNAI EGMORE – TIRUCHIRAPPALLI – MADURAI

Chennai Egmore – Tiruchi Rockfort Express	Chennai Egmore – Madurai Pandiyan Express	Chennai Egmore – Tiruchi Pallavan Express	Chennai Egmore – Madurai Vaigai Express	↓ TRAIN NAME ↑				Tiruchi – Chennai Egmore Pallavan Express	Madurai – Chennai Egmore Vaigai Express	Tiruchi – Chennai Egmore Rockfort Express	Madurai – Chennai Egmore Pandian Express
6177	2637	2605	2635	Train Number				2606	2636	6178	2638
22.30	21.30	15.30	12.25	km	d	Chennai Egmore	a	11.50	14.30	05.00	05.30
01.10	23.55	17.50	14.45	159	a	Villuppuram	d	09.15	11.55	01.50	02.25
01.20	00.05	17.55	14.50		d	Jn	a	09.10	11.50	01.40	02.20
02.03	00.48	18.38	15.38	214	a	Vriddhachalam	d	08.15	11.05	00.30	01.19
02.05	00.50	18.40	15.40		d	Jn	a	08.13	11.03	00.27	01.17
05.30	03.00	20.50	17.35	337	a	Tiruchirappalli	d	06.30	09.10	22.00	23.20
	03.05		17.40		d	Jn	a		09.05		23.15
	04.45		18.55	434	a	Dindigul Jn	d		07.45		21.55
	04.50		19.00		d		a		07.40		21.45
	06.30		20.15	496	a	Madurai Jn	d		06.45		20.30

a – Arrival Time d – Departure Time

The table given is the abstract taken from Table Number 71 of Southern Railway Time Table July 2003 – June 2004. Look at it now.

Let us go through the time table given above carefully.

1. The Table number is 71. (T. No. 71)
2. T.No.71 indicates the route ' Chennai Egmore – Tiruchirappalli – Madurai'.
3. Columns 1 to 4 indicate the number and name of the trains. They also indicate the departure and arrival time of the trains from various stations in the downward direction (↓) (From Chennai to Madurai). These trains are called down trains (DN).
4. Column 5 indicates various stations on the route. It also indicates the distance of each station from Chennai Egmore . From it we can calculate the distance between any two stations.
5. Columns 6 to 9 indicate the same informations in the upward direction (↑). (From Madurai to Chennai). These trains are called up trains (UP). Each train has two numbers, one while it travels in the downward direction and the other while it travels in the upward direction. For example the train number for Pallavan Express DN (↓) is 2605 and the train number for Pallavan Express UP (↑)is 2606.
6. All the trains mentioned in this table are Express trains.

Note: If we go through a Railway Time –table guide, we find many such time tables. There are different kinds of trains such as Passenger trains, Express trains, Shatabdi trains, Rajdhani Express trains and so on.

Note: In table No 71, all the trains starting from Chennai Egmore are Down trains (DN) and all the trains arriving at Chennai Egmore are up trains (UP).Using table. No. 71, we discuss the following examples.

Example 1:

(a) Name the stations between Chennai Egmore and Madurai.

(b) Find the distance between Villuppuram and Dindigul.

Solution:

(a) (Look at column 5. We find all stations between Chennai Egmore and Madurai where the Express train stops.)
Therefore the stations between Chennai Egmore and Madurai are Villuppuram, Vriddhachalam, Tiruchirappalli and Dindigul.

(b) (In the left edge of column 5 , we find the distance of various stations from Chennai Egmore.
Now we want to find the distance between Villuppuram and Dindigul. First note down the distance of Dindigul from Chennai Egmore. Then note down the distance of Villuppuram from Chennai Egmore. Subtracting we get the distance between Villuppuram and Dindigul.)

Distance from Chennai Egmore to Dindigul = 434 km
Distance from Chennai Egmore to Villuppuram = 159 km
∴ Distance between Villuppuram and Dindigul = 275 km

Example 2 :

(a) Mention the name of the train number 2636.

(b) What is the train number of 'Chennai Egmore – Madurai Pandian Express'?

Solution :

(Train numbers and Name of each train are given on the top of each column of the table.)

(a) The train number 2636 represents 'Madurai – Chennai Egmore Vaigai Express'.

(b) The train number of Chennai Egmore – Madurai Pandian Express is 2637 and it is the down train (↓) from Chennai Egmore to Madurai.

Example 3:

(a) At what time does '2635 Vaigai Express' arrive at Tiruchirappalli?

(b) How long does the train halt at Tiruchirappalli?

Solution :

First locate the column at which the arrival and departure time of '2635 Vaigai Express' is given , in table No. 71. It is in the fourth column.

(a) The arrival time at Tiruchirappalli is 17.35 hrs.

(b) To find the halting time subtract the arrival time from the departure time.

	hrs	mins
Departure time at Tiruchirappalli is	17	40
Arrival time at Tiruchirappalli is	17	35
∴ Halting time is	00	05

∴ The train halts at Tiruchirappalli for 5 minutes.

Note : If the journey crosses the midnight, we calculate the journey time in the following method.

1. Find the journey time from departure time till midnight (24.00 hrs or 00.00 hrs).
2. Find the journey time from midnight to arrival time .
3. The total journey time is the sum of the above two time.

Example 4 :

Find the journey time of Rockfort Express from Tiruchi to Chennai Egmore.

Solution :

Train number 6177 and train number 6178 both represent Rockfort Express. But we want to find the journey time from Tiruchirappalli to Chennai Egmore. Here the train to be considered is the UP train. Hence the train number is 6178.

Departure time from Tiruchi is 22.00 hrs

Arrival time at Chennai Egmore is 05.00 hrs.

Journey time from 22.00 hrs to mid night = 24.00 – 22.00

= 2 hrs

Journey time from midnight to arrival time = 5hrs

∴ Total journey time of Rockfort Express from Tiruchi to Chennai Egmore

= 2hrs + 5 hrs

= 7 hrs

Note: Express trains stop at some important railway stations only. But passenger trains stop at all the stations. Hence the passenger trains take more journey time than the express trains to travel equal distances. Considering the fares, passenger train fares are cheaper than the express train fares.

Example 5:

Train number 2675 Chennai – Kovai Express leaves Chennai central at 06.15 hrs and arrives Jolarpettai at 09.10 hrs. The train fare is Rs. 60.
Train number 101 Chennai – Bangalore passenger leaves Chennai at 04.30 hrs and arrives Jolarpettai at 13.00 hrs. The train fare is Rs. 33.
Find which is cheaper and which is faster.

Solution:

The train fare from Chennai central to Jolarpettai by Express train is	Rs. 60
The train fare from Chennai central to Jolarpettai by Passenger train is	Rs. 33
Difference is	Rs. 27

Hence travelling by passenger train is cheaper by Rs 27.

Now we calculate the journey time of both the trains.

Kovai Express	hrs	mins	
	08	70	
Arrival time at Jolarpettai is	~~09~~	~~10~~	(since 09.10 hrs = 08.70 hrs)
Departure time at Chennai – central is	06	15	
Journey time is	02	55	

∴ Journey time by Kovai Express is 2 hrs 55 mins.

Bangalore Passenger train	hrs	mins	
	12	60	
Arrival time at Jolarpettai is	~~13~~	~~00~~	(since 13.00 hrs = 12.60 hrs)
Departure time at Chennai - central is	04	30	
Journey time is	08	30	

∴ Journey time by Bangalore passenger train is 8 hrs 30 mins
The journey time by Kovai express is less than the journey time by Bangalore passenger train.
Hence travelling by Kovai express is faster.

Note: Travelling by express train costs more than the travelling by passenger train. But at the same time, the express train takes less time than the passenger train.

Exercise 3.1

1. Express in Railway Timings (24 Hour clock time): (Revision)

(a) 9.30 a.m(b) 12.00 noon (c) 4.10 p.m (d) Midnight

2. Express in ordinary timings : (Revision)

(a) 20.15 hrs (b) 08.40 hrs (c) 12.30 hrs (d) 10.35 hrs

3. A train departs from Chennai – central at 06.25 hrs and arrives Tirupathi at 09.30 hrs. Find the journey time.

T.No.38 MAYILADUTHURAI – KARAIKUDI – MAYILADUTHURAI

Tambaram Rameswaram Sethu Express	Mayiladuturai Karaikudi Passr.	TRAIN TIME				Rameswaram Tambaram Sethu Express	Karaikudi Mayiladuturai Passr.
6713	861	Train Number				6714	862
19.30 19.35	06.40	Km 0	a d	Mayiladuturai Jn	d a	00.55 00.50	20.15
	06.53	8	d	Manganallur H.	a		19.52
	07.06	16	d	Peralam	d		19.39
	07.13	23	d	Punthottam	d		19.31
	07.23	25	d	Nannilam	d		19.22
	07.44	36	d	Kodikkalpalayam H.	d		19.01
20.40 20.45	07.55 08.00	39	a d	Thiruvarur Jn.	d a	23.55 23.50	18.55 18.50
	08.25	49	d	Mavur Road	d		18.15
	08.32	53	d	Thirunellikaval	d		18.06
	08.38	55	d	Ammanur	d		18.00
	08.45	58	d	Alattambadi	d		17.55
	08.51	61	d	Manali H.	d		17.48
21.40 21.45	09.20 09.25	66	a d	Tiruturaipundi Jn.	d a	23.10 23.05	17.40 17.35
	09.45	80	d	Tillaivilagam	d		17.11
22.25	10.00	88	d	Mutupet	d	22.25	16.56
22.50	10.30	103	d	Adiramapattinam	d	21.55	16.29
23.15	10.50	115	d	Pattukottai	d	21.35	16.08
	11.25	136	d	Peravurani	d		15.20
00.25	12.10	161	d	Arantangi	d	20.25	14.50
01.05	12.45	182	d	Kandanur Puduvayal	d	19.55	14.15
01.55 02.00	13.20	187	a d	Karaikudi Jn.	d a	19.45 19.35	14.05

4. Read the Table No 38 and answer the questions given below:

(a) Indicate the route given in the table.

(b) Which are down (DN) trains? Which are up (UP) trains? (Write the train numbers only)

(c) How far is Karaikudi from Tiruturaipundi?

(d) At what time does the train No 6713 arrive at Thiruvarur? How long does it halt there?

5. Find the journey time of Train No. 6714 Sethu express and train No 862 Karaikudi – Mayiladuthurai passenger to reach Mayiladuthurai from Karaikudi and find which train takes more time.

3.1.3 Bus Time –table :

Travelling to all places by trains is impossible. Some remote places and villages cannot be reached by trains. Hence we need some other mode of transport. To reach the nook and corner of the villages the state government has developed bus facilities affordable to the common people. Travelling by bus has become very common. Bus time-tables are usually displayed at all important bus stations (bus stand). The bus time tables are simple and easy to read.

Bus time tables give the route, departure time, distance (in km) and fare (in Rs).

Note: The departure timings of buses and the fare do not remain always the same. These vary from time to time.

Example 6:

Read the bus time – table given below and answer the questions that follow:

State Express Transport Corporation – Chennai.

Route	Via	Dep.time from Chennai	Dep.time from other direction	Dist (km)	Fare (Rs)
Chennai – Coimbatore	Salem , Erode	19 .30 hrs	19 .30 hrs	510	169
Chennai – Kanyakumari	Trichy, Madurai	13 .45 hrs	14 .30 hrs	709	232
Chennai – Nagercoil	Trichy , Madurai	16 .45 hrs	16 .15 hrs	689	226
Chennai – Tirunelveli	Trichy, Madurai	19 .00 hrs	19 .00 hrs	609	200
Chennai – Tuticorin	Trichy, Madurai	17 .30 hrs	17 .00 hrs	592	195
Chennai – Madurai	Villuppuram , Trichy	20.45 hrs	21.45 hrs	450	149

(a) What is the title of the bus time table?
(b) How many routes are listed in the time table?
(c) At what time does the bus for Nagercoil leave Chennai?
(d) At what time does the bus for Chennai leave Tuticorin?
(e) How far is Kanyakumari from Chennai?
(f) What is the fare from Chennai to Coimbatore?
(g) Through which towns do the bus for Nagercoil pass?

Solution:

(a) The title of the table is State Express Transport Corporation, Chennai.
(b) There are 6 routes listed in the time table.
(c) The bus for Nagercoil leaves Chennai at 16.45 hrs i.e 4.45p.m.
(d) The bus for Chennai leaves Tuticorin at 17.00 hrs i.e 5.00p.m.
(e) Kanyakumari is at a distance of 709 km from Chennai.
(f) The fare from Chennai to Coimbatore is Rs 169.
(g) The bus for Nagercoil passes through Trichy and Madurai.

Example 7:

From the table given below, find out which route should be taken to reach Coimbatore at about 5.00 a.m ? which journey is faster?

Route	Dep.from Chennai	Arr.at Coimbatore
SETC BUS	19.30 hrs	07.00 hrs
Nilgris Express	20.30 hrs	04.45 hrs

Solution:

Though the bus departs from Chennai one hour earlier than the train, it reaches Coimbatore only at 07.00 hrs i.e 7.00 a.m But the train reaches Coimbatore at 04.45 hrs. (i.e) 4.45 a.m. Hence train route should be taken to reach Coimbatore in time (i.e) at about 5.00 a.m Now we calculate the journey time by bus and by train.

SETC BUS:

Departure time from Chennai is 19.30 hrs

Arrival time at Coimbatore is 07.00 hrs

Journey time from 19.30 hrs to 24.00 hrs

	hrs	mins
	23	60
=	~~24~~	~~00~~
	−19	30
=	4	30

Journey time from midnight to 07.00 hrs = 7hours.

∴ Total journey time = 4 hrs 30 mins + 7 hrs.

	hrs	mins
	4	30
+	7	00
=	11	30

∴ Journey time by SETC bus is 11 hrs 30 mins.

Nilgris Express Train :

Departure time from Chennai is 20.30 hrs.

Arrival time at Coimbatore is 04.45 hrs

Journey time from 20.30 hrs to midnight

	hrs	mins
	23	60
=	~~24~~	~~00~~
−	20	30
=	3	30

Journey time from midnight to 04.45 hrs = 4 hrs 45 min

∴ Total journey time = 3 hrs 30 mins + 4 hrs 45 mins

+	4	45
	7	75 (60 +15)

= 8 hrs 15 mins

∴ Journey time by the Nilgris Express is 8 hrs 15 mins.

The journey time taken by the bus is more than the journey time taken by the train. Hence the train journey is faster than the bus journey.

Exercise 3.2

1. Read the bus time table given below and answer the questions that follow:

ROUTE NO : 137

CHENNAI - MADURAI VIA – VILLUPPURAM , TRICHY

STATION	*KM*	*ARR*	*DEP*
Chennai	*0*	–	*20.45*
Tindivanam	*125*	*23.15*	*23.30*
Villuppuram	*162*	*00.15*	*00.20*
Thozhudur	*245*	*02.00*	*02.15*
Trichy	*322*	*03.45*	*04.00*
Madurai	*450*	*06.40*	–

(a) What is the route number and route of the time table?

(b) What are the stations given in between Chennai and Madurai?

(c) How far is Trichy from Tindivanam?

(d) How long does the bus halt at Tindivanam?

(e) Find the journey time from Chennai to Madurai?

2. Read the following table and answer the questions given below:

Route No	*Route*	*Journey time*	*Distance (km)*
170	*Chennai – Tuticorin*	*13 hrs 30 mins*	*592*
123	*Chennai – Trichy*	*7 hrs*	*322*
460	*Chennai –Coimbatore*	*11 hrs 30 mins*	*510*
282	*Chennai – Kanyakumari*	*16 hrs*	*709*
137	*Chennai – Madurai*	*10 hrs*	*450*

(a) Find the journey time by Route no: 123 from Chennai to Trichy.

(b) What route does the route number 137 indicate?

(c) SETC bus departs from Chennai at 7.00 a.m . When does it arrive at Tuticorin?

(d) Route number 282 Chennai – Kanyakumari bus passes through Madurai. Find the journey time from Madurai to Kanyakumari. (Hint : It takes 10 hrs to travel from Chennai to Madurai).

(e) Which town is nearer to Chennai and which town is far off from Chennai?

3. From the time table given below find which journey is faster. Find whether the quicker journey is also the cheaper journey.

Route	*Departure from Chennai*	*Arrival at Trichy*	*Fare from Chennai*
Pallavan Express Train	*15.30 hrs*	*20.50hrs*	*Rs. 136*
SETC Bus	*13.45 hrs*	*20.45hrs*	*Rs. 109*

3.2 Area and Perimeter :

We have already learnt the concept of area and perimeter and their units. We have also learnt the formulae for finding the area and the perimeter of simple closed figures like rectangle, square and right triangle. In this lesson we learn about the area of some more simple closed figures such as triangle, parallelogram, rhombus, trapezium, circle and sectors.

3.2.1 Revision
3.2.2 Area of a Triangle
3.2.3 Area of a Parallelogram
3.2.4 Area of a Rhombus
3.2.5 Area of a Trapezium
3.2.6 Perimeter and area of Circles and Sectors
3.2.7 Area of Pathways

3.2.1 Revision :

We now recall what we have studied about the area and perimeter of rectangle, square and right triangle.

Area :

The amount of surface occupied by a plane figure is called its area. The unit of area is square units.

Perimeter:

The length of the boundary of any closed figure is called its perimeter. The unit of perimeter is units.

Rectangle :

1. Area , A = lb sq. units

2. Perimeter, P = 2($l + b$) units where l - length , b - breadth

Square :

1. Area , A = a^2 sq. units

2. Perimeter, P = 4a units where a – side

Right triangle :

1. Area , A = $\frac{1}{2}bh$ sq. units where, b – base , h – vertical height

2. Perimeter, P = Sum of the measures of its three sides

Example 8:

Find the area and perimeter of a rectangle with length 7.5 cm and breadth 4.4 cm

Solution:

Given l = 7.5 cm and b = 4.4 cm

Area , A = lb sq .units

= 7.5×4.4

= 33

$\therefore$ Area = 33 cm^2

Perimeter , P = 2 ($l + b$) units

= 2 (7.5 + 4.4)

= 2×11.9

$$= 23.8$$

$$\therefore \text{Perimeter} = 23.8 \text{ cm}$$

Example 9:

Find the area of a square whose perimeter is 44 m.

Solution:

Given Perimeter of the square , $P = 44$ m

We know that $P = 4a$ units

i.e $4a = P$

$$a = \frac{P}{4}$$

$$\therefore \text{ side of the square, } a = \frac{44}{4} = 11$$

$$a = 11\text{m}$$

Area , $A = a^2$ sq. units

$$= 11 \times 11$$

$$= 121$$

$$\therefore \text{Area} = 121 \text{ m}^2$$

Example 10:

The base and height of a right triangular ground are 60 m and 45 m. Find the cost of levelling the ground at Rs 150 per are [1 Are = 100 sq. m].

Solution:

Given base , $b = 60$m ; height , $h = 45$m

Cost of levelling 1 are = Rs. 150

$$\text{Area of the right triangle , } A = \frac{1}{2}bh \text{ sq. units}$$

$$= \frac{1}{2} \times 60 \times 45$$

$$= 1350\text{m}^2$$

$$100\text{m}^2 = 1 \text{ are}$$

$$\therefore 1350\text{m}^2 = \frac{1350}{100} = 13.5 \text{ are}$$

Cost of levelling 1 are = Rs. 150

$\therefore$ Cost of levelling 13.5 ares = Rs. 13.5×150 = Rs.2,025

$\therefore$ Cost of levelling the right triangular ground = Rs. 2,025

Example 11 :

The side of a square hall is 6m. It is to be covered with rectangular tiles of size 25 cm × 12 cm. Find the total cost of the tiles at the rate of Rs 20 per tile.

Solution:

Side of the square Hall, $a = 6\text{m} = 600$ cm (why?)

Area , $A = a^2$ sq. units

$$= 600 \times 600$$

$$= 360000 \text{ cm}^2$$

length of one tile , $l = 25$cm ; breadth , $b = 12$ cm

$$\text{Area of one tile, A} = lb \text{ sq units}$$
$$= 25 \times 12$$
$$= 300 \text{ cm}^2$$
$$\text{Number of tiles required} = \frac{\text{Area of the hall}}{\text{Area of one tile}}$$
$$= \frac{360000}{300}$$
$$= 1200 \text{ tiles}$$
$$\text{Cost of 1 tile} = \text{Rs. } 20$$
$$\therefore \text{ Cost of 1200 tiles} = 1200 \times 20$$
$$= \text{Rs. } 24{,}000$$
$$\therefore \text{ The total cost of the tiles} = \text{Rs. } 24{,}000$$

Exercise 3.3

1. ***(a) Area of a rectangle is _____ sq. units***
   (b) Perimeter of a rectangle is _____ units.
   (c) Area of a square is _____sq.units.
   (d) Perimeter of a square is _____ units
   (e) Area of a right triangle is _____sq. units.
2. ***Find the area and perimeter of a rectangle having***
   (a) length = 15m, breadth = 8m
   (b) length = 160cm, breadth = 1.2m
3. ***Find the area and perimeter of square having***
   (a) side = 24cm (b) side = 1.5m
4. ***Find the area of a right triangle having***
   (a) base = 120mm; height = 10cm
   (b) base = 1.6m; height 2m
5. ***The perimeter of a rectangular field is 340m and its length is 120m. Find the area of the field in ares (1are = 100m^2).***
6. ***A hall 24m long and 12m broad is to be fixed with square tiles of side 1.2m. Find the number of tiles required.***

3.2.2 Area of a Triangle :

We have learnt that the area of a right triangle is half the area of the rectangle.

The area of a right triangle, A = $\frac{1}{2}$ *bh* sq.units. In this chapter we learn about the area of any triangle.

3.2.2 (a) To find the area of a triangle :

Take a rectangular piece of paper. Name the vertices as A, B, C, and D. Now mark any point E on DC. Join A and E. Join B and E. We get a triangle ABE inscribed in the rectangle ABCD (see fig. 3.1)

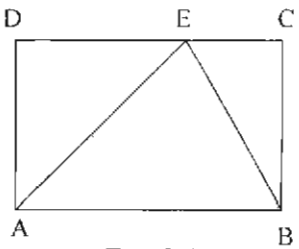


Fig. 3.1

Now mark another point F on AB such that DE = AF. Join E and F as shown in fig.3.2. We observe that BC and FE are of same length and FE is called the height (or) the altitude of the triangle ABE. Let FE be 'h' units. Now the rectangle ABCD has four parts namely (1) , (2) , (3) and (4).

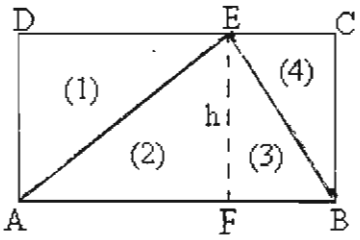


Fig. 3.2

Cut the rectangle along AE and BE. We get triangle (1) i.e. AED and triangle (4) i.e. BCE separately (see figure 3.3).
Now fit the part (1) on part (2) and the part (4) on part (3)
We see that they fit on each other exactly.

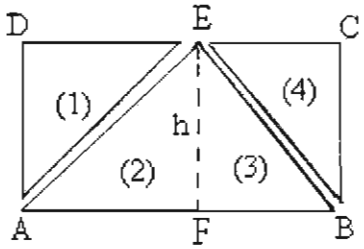


Fig . 3.3

Area of triangles (1) + (2) + (3) + (4) = Area of rectangle ABCD

Here Area of triangle (1) = Area of triangle (2)

$$\therefore \text{ Area of triangle (2)} = \frac{\text{Area of rectangle AFED}}{2}$$

Also area of triangle (4) = Area of triangle (3)

$$\therefore \text{ Area of triangle (3)} = \frac{\text{Area of rectangle FBCE}}{2}$$

Area of triangle (2) + Area of triangle (3) = [$\frac{1}{2}$ area of rectangle AFED + $\frac{1}{2}$ Area of rectangle FBCE]

Area of Δ ABE = $\frac{1}{2}$ [Area of rectangle AFED + Area of rectangle FBCE]

$= \frac{1}{2} \times$ Area of rectangle ABCD

$= \frac{1}{2} \times bh$ [b – length ; h – breadth]

$\therefore$ Area of triangle ABE $= \frac{1}{2}$ bh sq.units (see fig. 3.4)

(Where b is the base and h is the height)

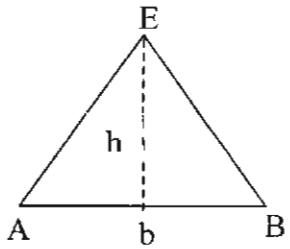


*Fig.*3.4

$\therefore$ **Area of a triangle** $= \frac{1}{2} \times bh$ **sq.units**

Where b **= base of the triangle,** h **= height of the triangle**

Example 12:

Find the area of triangle ABC in fig. 3.5

Solution:

In fig.3.5,

AB = b = 16 cm and CD = h = 10 cm

Area of the triangle , $A = \frac{1}{2} bh$ sq.units

$= \frac{1}{2} \times 16 \times 10$

$= 80$

$\therefore$ Area of the triangle $= 80cm^2$

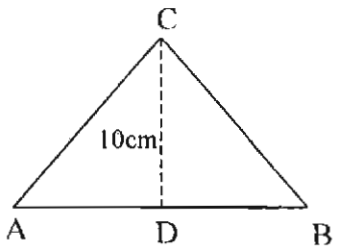


Fig. 3.5

Example 13 :

The area of a triangular field is $1200m^2$. The base of the triangular field is 60m. Find the height.

Solution:

Given area , $A = 1200m^2$ and b = 60m

Area of the triangle , $A = \frac{1}{2} bh$ sq.units

We can write the equation as $\frac{1}{2} bh = A$

$\frac{1}{2} \times 60 \times h = 1200$

$30 \times h = 1200$

$h = \frac{1200}{30}$

$h = 40$

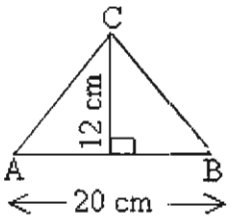
$\therefore$ Height of the triangular field = 40m

Discuss: In a triangle 'b' represents base and 'h' the height of the triangle

What do 'b' and 'h' represent in a right triangle?

Exercise 3.4

1. ***Find the area of the triangle ABC in the figure given below:***



2. ***Find the area of the triangle whose***

   (a) base = 16cm, height = 7.5cm ***(b) base = 4.8m, height = 25dm***

   (c) base = 6cm, height = 35mm

3. ***Find the base of the triangle whose***

   (a) area = 2400 cm^2, height = 8dm ***(b) area = 120m^2, height = 16m***

4. ***Find the height of triangle whose***

   (a) area = 14 cm^2, base = 4cm ***(b) area = 30dm^2, base = 6dm***

5. ***A field is in the form of a triangle. Its base is 160m and height 200m. Find the area of the field in hectares (1 hectare = 10000m^2)***

3.2.3 Area of a Parallelogram :

In a quadrilateral if both the pairs of opposite sides are parallel then it is a parallelogram (see fig.3.6). Here the sides AB and DC are parallel and sides AD and BC are parallel. We use the symbol '||' which denotes " is parallel to" (i.e.) AB is parallel to DC and can be written as AB|| DC and AD is parallel to BC and can be written as AD || BC.

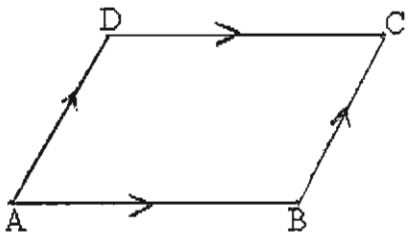


Fig. 3.6

3.2.3 (a) Base and altitude of a parallelogram :

In a parallelogram any side may be called its base. But for convenience we consider AB (Fig.3.7) as the base (b) of the parallelogram. Here DE is the line segment drawn perpendicular to AB. (i.e.) DE is the perpendicular distance between the sides AB and DC and we call it as the **altitude** or **height** (h) corresponding to the base AB.

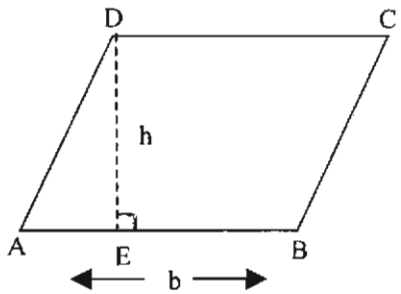


*Fig.*3.7

Note : The perpendicular distance between the opposite sides of the parallelogram is the height (*h*)

3.2.3 (b) To find the area of a parallelogram :

Consider a parallelogram ABCD (Fig.3.8) with base 'b' units and altitude (height)'h' units.
Draw the diagonal BD. Cut the parallelogram along BD. We get two triangles ABD and BCD.
Now fit the triangle BCD on the triangle DAB. We find that they fit on each other exactly.
From this we come to a conclusion that the diagonal of the parallelogram divides it into two triangles equal in area.

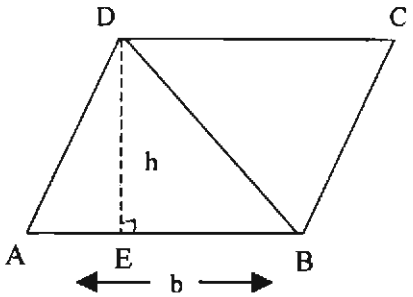


Fig. 3.8

$\therefore$ Area of the parallelogram ABCD = 2 × Area of triangle ABD

$$\text{Area of } \Delta ABD = \frac{1}{2} \times \text{base} \times \text{height sq. units}$$

$$= \frac{1}{2} bh \text{ sq. units.}$$

$$\therefore \text{Area of the parallelogram ABCD} = 2 \times \frac{1}{2} bh \text{ sq. units.}$$

$$= bh \text{ sq. units.}$$

$\therefore$ **Area of a parallelogram , A = *bh* sq. units.**

Where *b* = base of the parallelogram

h **= altitude of the parallelogram**

Example 14 :

Find the area of a parallelogram whose base is 1.5dm and the altitude is 8cm

Solution :

Given b = 1.5dm or 15cm and h = 8cm

Area of the parallelogram, A = $b \times h$ sq.units

$= 15 \times 8$

$= 120$

$\therefore$ Area of the parallelogram = $120cm^2$

Example 15:

Find the altitude of a parallelogram whose area is $5400cm^2$ and the base is 45cm

Solution:

Given area, A = $5400cm^2$ and b = 45cm

$$\text{Altitude } h = \frac{A}{b} \text{ units}$$

$$= \frac{5400}{45}$$

$$= 120$$

$\therefore$ Altitude = 120cm

Exercise 3.5

1. Find the area of the following parallelograms

(a) base = 55cm and altitude = 40cm

(b) base = 42 dm and height = 25 cm

(c) base = 3 m 2 dm and altitude = 20 dm

2 Find the base of a parallelogram whose area 720m^2 and the altitude is 20 m.

3.Find the altitude of a parallelogram whose area is 180 sq.cm and the base is 18 cm

4. A field in the form of a parallelogram has base 15 dam and altitude 8 dam. Find the cost of levelling the field at the rate of Rs. 25 per are.

5. A ground is in the form of a parallelogram. Its base is 200 m and its altitude is 150 m. Find the area of the ground in hectares.

3.2.4 Area of Rhombus :

In a parallelogram if all the four sides are equal then it is called rhombus. (See fig.3.9) If the base of the rhombus is 'b' units and its corresponding altitude is h units, then the area of the rhombus is given by

A = *bh* sq.units .

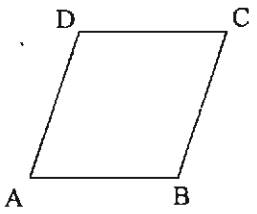


Fig. 3.9

Discuss : Can you prove that the area of the rhombus A = b × h sq.units using the same principle used to find the area of the parallelogram?

3.2.4 (a) The diagonals of a rhombus bisect at right angles :

Consider the rhombus ABCD (Fig.3.10). Draw the diagonals AC and BD. Let them intersect at O. Measure $\angle AOB$, $\angle BOC$, $\angle COD$ and $\angle DOA$.. We can find that each angle measures 90°. From this we observe that diagonals of the rhombus are perpendicular to each other.

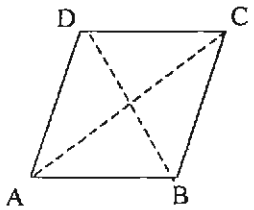


Fig. 3.10

Now measure the lengths of the segments OA and OC of the diagonal AC. We find that $\overline{OA} = \overline{OC}$. Measure the length of the segments OB and OD of the diagonal BD. Here also $\overline{OB} = \overline{OD}$. From this we come to a conclusion that the diagonals of the rhombus bisect each other.

Hence the diagonals of a rhombus bisect at right angles.

3.2.4 (b) The diagonals of a rhombus divide the rhombus into four right triangles :

Now cut the rhombus along the diagonals AC and BD. We get four right triangles AOB, BOC, COD and DOA (see fig.3.11). By fitting the triangles one above the other we can see that all the triangles fit on each other exactly.

Hence the diagonals of rhombus divide the rhombus into four right triangles equal in area.

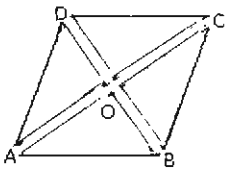


Fig. 3.11

3.2.4 (c) To find the area of a rhombus in terms of its diagonals :

Consider the rhombus ABCD (Fig3.12). Diagonal AC = d_1 units and diagonal BD = d_2 units.

We know that the diagonals of a rhombus bisect each other at right angles.

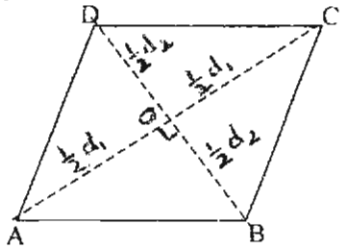


Fig. 3.12

$\therefore$ OA = $\frac{1}{2}$ d_1 , OB = $\frac{1}{2}$ d_2 and $\angle AOB = 90°$

OA and OB are the sides containing the right angle

$\therefore$ Area of right triangle AOB = $\frac{1}{2}$ × OA × OB sq. units

$$= \frac{1}{2} \times \frac{1}{2} d_1 \times \frac{1}{2} d_2$$

$$= \frac{1}{8} \times d_1 \times d_2 \text{ sq. units}$$

We know that the diagonals of a rhombus divide it into four right triangles equal in area.

$\therefore$ Area of rhombus ABCD = 4 × area of Δ AOB sq. units.

$$= 4 \times (\frac{1}{8} \times d_1 \times d_2) = \frac{1}{2} d_1 d_2 \text{ sq. units}$$

$\therefore$ **Area of a rhombus , A = $\frac{1}{2}$ d_1 d_2 sq. units**

(i.e) Area of a rhombus = $\frac{1}{2}$ × [product of the diagonals] sq. units

Example 16:

One side of a rhombus is 7.5 cm and the altitude is 4cm. Find the area of the rhombus .

Solution:

Given base (side), b = 7.5 cm; h = 4cm

Area of the rhombus , A = bh sq. units

= 7.5 × 4

= 30

$\therefore$ Area of the rhombus = 30 sq. cm

Example 17:

Find the area of a rhombus whose diagonals are 15cm and 12 cm .

Solution:

Given d_1 = 15cm , d_2 = 12 cm

Area of the rhombus , A = $\frac{1}{2}$ d_1 d_2 sq. units

$$= \frac{1}{2} \times 15 \times 12$$
$$= 90$$

$\therefore$ Area of the rhombus = $90cm^2$

Example 18 :

Area of a rhombus is 100sq.cm. One of its diagonals is 16cm. Find the length of the other diagonal.

Solution :

Given area, A = 100sq.cm ; diagonal d_1 = 16cm

Area, $A = \frac{1}{2} d_1 d_2$ sq. units

$$(\text{i.e}) \frac{1}{2} \times d_1 \times d_2 = 100$$
$$\frac{1}{2} \times 16 \times d_2 = 100$$
$$8 \times d_2 = 100$$
$$d_2 = 100 \times \frac{1}{8}$$
$$= 12.5$$

$\therefore$ The length of the other diagonal = 12.5cm

Note: We know all the four sides of a square are equal. Therefore a square is a rhombus. Hence we can use the formula $A = \frac{1}{2} \times d_1 \times d_2$ sq.units. to find the area of a square when the diagonals are given.

Discuss: A square is a rhombus but a rhombus is not a square. Why?

Exercise 3.6

1. ***Find the area of a rhombus whose side is 24 cm and the altitude (height) is 20 cm.***
2. ***Find the area of the rhombus whose diagonals are***
   (a) 20cm, 16cm (b) 10.5cm, 1.6dm (c) 6m, 10.2m
3. ***The area of a rhombus is 25 sq.m. If one diagonal is 10 m long, find the length of the other diagonal.***
4. ***A field is in the form of a rhombus. The diagonals of the field are 60 m and 80 m. Find the cost of levelling it at the rate of Rs. 2 per sq.m.***
5. ***Area of a rhombus is 5 ares. The length of one diagonal is 25 m. Find the length of the other diagonal.***

3.2.5 Area of a Trapezium :

We know that in a quadrilateral if one pair of opposite sides are parallel then the quadrilateral is known as a trapezium (See fig.3.13). Here the sides AB and DC are parallel. The sides AD and BC are not parallel.

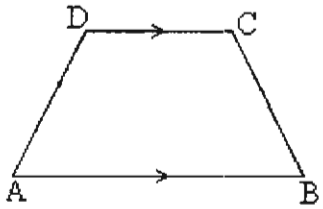


*Fig.*3.13

Consider the trapezium ABCD (Fig. 3.14). The side AB is parallel to the side DC (i.e.) AB || DC.

Let AB = a units and DC = b units.

Join the diagonal BD. Draw DE perpendicular to AB. Also draw BF perpendicular to DC (produced).

Here ED = BF = the height of the trapezium = h units

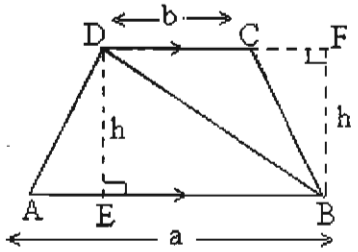


*Fig.*3.14

3.2.5 (a) To find the area of a trapezium :

Area of the trapezium ABCD = Area of triangle ABD + Area of triangle BCD

$$= \left(\frac{1}{2} \times AB \times ED\right) + \left(\frac{1}{2} \times DC \times BF\right)$$

$$= \left(\frac{1}{2} \times a \times h\right) + \left(\frac{1}{2} \times b \times h\right)$$

$$= \left(\frac{1}{2}ah + \frac{1}{2}bh\right)$$

$$= \frac{1}{2}(a + b) \times h \text{ sq .units}$$

$\therefore$ Area of a trapezium, A $= \frac{1}{2}(a + b) \times h$ sq. units.

(i.e) Area of a trapezium $= \frac{1}{2} \times$ (sum of the parallel sides × Perpendicular distance between the parallel sides) sq. units.

Example 19 :

The parallel sides of a trapezium are 12 cm and 8 cm. The perpendicular distance between them is 9 cm. Find the area of the trapezium.

Solution :

Given a = 12 cm and b = 8 cm (parallel sides)

h = 9 cm (distance between the parallel sides)

$$\text{Area of the trapezium, } A = \frac{1}{2}(a + b) \times h \text{ sq.units}$$

$$= \frac{1}{2} \times (12 + 8) \times 9$$

$$= \frac{1}{2} \times 20 \times 9 = 90$$

$\therefore$ Area of the trapezium = 90 cm^2

Example 20 :

Area of a trapezium is 180 cm^2. The parallel sides are 20 cm and 16 cm. Find the distance between the parallel sides.

Solution :

Given A = 180 cm^2, a = 20 cm and b = 16 cm

Area of the trapezium, $A = \frac{1}{2}(a+b) \times h$ sq.units

(i.e) $\frac{1}{2}(a+b) \times h = A$

$\frac{1}{2}(20+16) \times h = 180$

$\frac{1}{2} \times 36 \times h = 180$

$18 \times h = 180$

$h = \frac{180}{18}$

$h = 10$ cm

∴ The distance between the parallel sides = 10 cm

Exercise 3.7

1. ***The parallel sides and the perpendicular distance between the parallel sides of some trapeziums are given below. Find their areas.***
   - ***(a) Parallel sides 12.2 cm and 8.8 cm and height 6 cm***
   - ***(b) Parallel sides 2.5 dm and 1.3 dm and height 80 cm***
   - ***(c) Parallel sides 15 cm and 9 cm and height 5 cm***
2. ***The area of a trapezium is 138m^2. The parallel sides are 15 m and 8 m. Find the perpendicular distance between them.***
3. ***The parallel sides of the trapezium are 7.5 cm and 12.5 cm. The area of the trapezium is 65 cm^2. Find the perpendicular distance between the parallel sides.***
4. ***The parallel sides of a trapezium are 8 m and 40 dm. The perpendicular distance between parallel sides is 25 dm. Find the area of the trapezium.***
5. ***A garden is in the form of a trapezium. The parallel sides are 30 m and 24 m. The perpendicular distance between the parallel sides is 20 m. Find the area in ares.***

3.2.6 Perimeter and area of Circles and Sectors :

We have already learnt about the circle in the previous standard . Can you give a few examples for circular objects ?

One rupee coin, bangles, wheel of a bicycle are a few examples for circular objects. Draw a circle of 5 cm radius using a compass. Mark the points as shown in figure 3.15

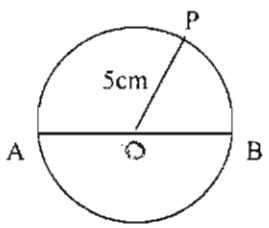


Fig. 3.15

In the figure, O is the centre of the circle. OP is a radius of the circle. AB is a diameter of the circle. If we take any point on the circle the distance between that point and the centre is 5 cm. Hence all the points on the circle are equidistant from the centre of the circle.

Therefore **a circle can be defined as the path traced by a variable point which is equidistant from the fixed point.** The fixed point is the centre of the circle and the equal distance is the radius of the circle. A line segment joining the centre of the circle and any point on the circle is called a **radius** of the circle. Radius is usually denoted by '*r*'. A line segment joining any two points on the circle is called a **Chord**. **Diameter** is a chord passing through the centre of the circle. Diameter is usually denoted by '*d*'.

Discuss : What is the longest chord ?

Relation between Diameter and Radius:

Diameter = 2 × radius

(i.e) $d = 2r$

3.2.6 (a) Circumference of a circle :

The perimeter of a circle is called the circumference of the circle.

Relation between diameter and circumference of a circle:

Consider a circle of diameter 'd'. Take a thread and fix the thread around the circumference of the circle. Measure the length of this thread using a scale. Let it be 'C'. Find out the ratio $\frac{C}{d}$. Repeat this experiment with some more circles and find the ratio $\frac{C}{d}$ in each case.

Sl.NO.	Radius (r) cm	Circumference (C) cm	Diameter (d) cm	$\frac{\text{Circumference}}{\text{diameter}} = \frac{C}{d}$
1	7	44	14	$\frac{44}{14} = \frac{22}{7}$
2	14	88	28	$\frac{88}{28} = \frac{22}{7}$
3	10.5	66	21	$\frac{66}{21} = \frac{22}{7}$
4	21	—	—	—
5	3.5	—	—	—

Table 3.1

From the table 3.1 we can observe that the ratio $\frac{C}{d}$ is a constant and equal to $\frac{22}{7}$ for all the circles. This constant is denoted by the Greek letter π and read as pi.

∴ The relation between the circumference and the diameter of a circle is

$$\frac{C}{d} = \frac{\text{Circumference}}{\text{diameter}} = \pi$$

or $C = \pi d$

$$= \pi \times 2r \text{ (since } d = 2r \text{)}$$

$$C = 2\pi r \text{ units}$$

$\therefore$ The circumference of a circle, $C = 2\pi r$ units

Example 21 :

Find the circumference of a circle whose radius is 7 cm.

Solution :

$$\text{Radius, r} = 7 \text{ cm}$$

$$\text{Circumference, C} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 7 = 44$$

$\therefore$ Circumference of the circle $= 44$ cm

Example 22:

Find the radius of the circle whose circumference is 66 cm.

Solution:

$$\text{Circumference , C} = 66 \text{ cm}$$

$$C = 2\pi r$$

$$\therefore 2\pi r = C$$

$$2 \times \frac{22}{7} \times r = 66$$

$$\therefore r = \frac{66 \times 7}{2 \times 22}$$

$$= 10.5$$

$\therefore$ Radius of the circle $= 10.5$ cm

Example 23:

The diameter of a wheel is 3.5 m. Find out the distance travelled when it completes 100 revolutions.

Solution:

$$\text{Diameter , } d = 3.5 \text{ m}$$

$$\text{Circumference , C} = \pi d$$

$$= \frac{22}{7} \times 3.5$$

$$= 11 \text{ m}$$

Distance travelled for one complete revolution $= 11$ m

Distance travelled for 100 complete revolutions $= 11 \times 100 = 1100$ m

$\therefore$ Distance travelled for 100 revolutions $= 1100$ m

Example 24 :

A bicycle wheel makes 50 revolutions to cover a distance of 6600 cm. Find the radius of the wheel.

Solution : Circumfernce, C = $\dfrac{\text{Distance travelled}}{\text{Number of revolutions}}$

$$C = \frac{6600}{50} = 132$$

$$C = 2\pi r = 132$$

$$2 \times \frac{22}{7} \times r = 132$$

$$r = \frac{132 \times 7}{2 \times 22}$$

$$= 21$$

$\therefore$ Radius of the bicycle wheel = 21cm

Example 25:

The diameter of a wheel is 28 m. How many revolutions does it make in travelling a distance of 2.2 km ?

Solution:

Diameter , d = 28 m

Circumference , $C = \pi d$

$$= \frac{22}{7} \times 28$$

$$= 88 \text{ m}$$

$$\text{Number of revolutions} = \frac{\text{Distance travelled}}{\text{Circumference}}$$

$$= \frac{2200}{88} \text{ (since 2.2 km = 2200 m)}$$

$$= 25$$

$\therefore$ Number of revolutions = 25

Exercise 3.8

1. ***Find the circumferences of circles whose radii are***
   (a) 14 cm (b) 28 cm (c) 4.9 m
2. ***Find the circumferences of circles whose diameters are***
   (a) 35 cm (b) 42 dm (c) 6 .3 m
3. ***Find the radii of the circles whose circumferences are***
   (a) 440 mm (b)132 cm (c)52.8 m
4. ***Find the diameters of the circles whose circumferences are***
   (a) 572 cm (b) 77 m (c)π m
5. ***The radius of a wheel is 3.5 cm. If the wheel makes 50 revolutions find the distance covered .***
6. ***The radius of a circular park is 70 m. Find the cost of fencing the park at the rate of Rs. 5 per metre .***
7. ***A wire of length 88 cm is bent as a circle. What is the radius of the circle?***
8. ***The cost of fencing a circular park at the rate of Rs 2 per metre is Rs. 1056. What is the radius of the park?***

9. ***The diameter of a wheel is 21 cm. Find out the distance travelled when it completes 50 revolutions.***

10. ***A wheel makes 10 revolutions to cover a distance of 4400 cm. Find the diameter of the wheel.***

3.2.6 (b) Area of a circle :

Draw any circle. Divide it into 16 equal parts as shown in the fig.3.16. Cut and paste them as shown in the fig.3.17. Now we get a parallelogram.

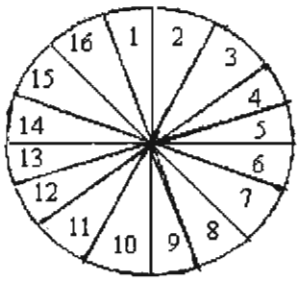


*Fig.*3.16

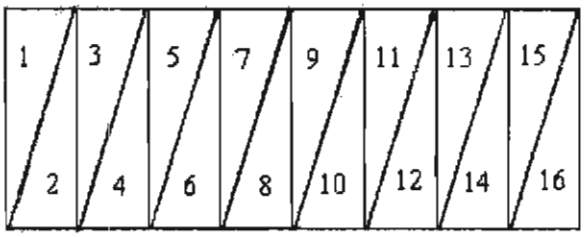


Fig. 3.17

If we divide the circle into more number of equal parts and rearrange them, we will get approximately a rectangle. Isn't it? So as we increase the number of divisions of the circle, the parallelogram becomes a rectangle. From the figure, we find that the two longer sides (length) of the rectangle is the circumference of the circle and the smaller side (breadth) of the rectangle is the radius of the circle.

$$\therefore \text{ The length of the rectangle } = \frac{\text{Perimeter of the circle}}{2}$$

$$= \frac{2\pi r}{2}$$

$$= \pi r$$

$\therefore$ The breadth of the rectangle = radius of the circle

$$= r$$

$\therefore$ Area of the circle = Area of the rectangle

$$= \textit{length} \times \textit{breadth}$$

$$= \pi r \times r$$

$$= \pi r^2$$

$\therefore$ **Area of a circle = πr^2 sq. units**

Discuss : If 'r' is the radius then the area of the circle is πr^2 sq. units.

If 'd' is the diameter then the area of the circle is $\pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$ sq .units.

Example 26 :

Find the area of the circle whose radius is 7cm

Solution:

Radius, r = 7cm

$$\text{Area of the circle} = \pi r^2 \text{ sq. units}$$
$$= \frac{22}{7} \times 7 \times 7$$
$$= 154$$
$$\therefore \text{ Area of the circle} = 154 \text{ sq.cm.}$$

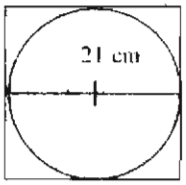
Example 27:

Find the area of the largest circle that can be inscribed in a square of side 21 cm .

Solution :

From the fig. 3.18, diameter of the largest circle, $d = 21$cm

$$\text{Area of the largest circle} = \frac{\pi d^2}{4} \text{ sq. units.}$$
$$= \frac{22}{7} \times \frac{1}{4} \times 21 \times 21$$
$$= 346.5$$
$$\therefore \text{ Area of the largest circle} = 346.5 \text{ sq. cm}$$



21 cm

Fig. 3.18

Example 28 :

Find the area of the circle whose circumference is 88 cm.

Solution :

$$\text{Circumference of the circle , C} = 2\pi r \text{ units}$$
$$= 88\text{cm}$$
$$2 \times \frac{22}{7} \times r = 88$$
$$r = 88 \times \frac{7}{22} \times \frac{1}{2}$$
$$= 14$$
$$\therefore \text{ Radius} = 14 \text{ cm}$$
$$\text{Area of the circle} = \pi r^2 \text{ sq. units}$$
$$= \frac{22}{7} \times 14 \times 14 = 616$$
$$\therefore \text{ Area of the circle} = 616 \text{ sq.cm}$$

Exercise 3.9

1. ***Find the areas of the circles whose radii are***
   (a) 21 cm (b) 3.5 cm (c) 14 dm
2. ***Find the areas of the circles whose diameters are***
   (a) 14 m (b) 10.5 cm (c) 35 dm
3. ***Find the area of the largest circle that can be inscribed in a square of side 4.2 cm.***
4. ***If the largest possible circle is cut off from a square sheet of side 21 cm, what is the area of the remaining piece of sheet ?***
5. ***What is the area of the circle whose perimeter is 22 m?***
6. ***The cost of fencing a circular park at the rate of Rs. 5 per metre is Rs. 220. Find the area of the park .***

7. ***Find the radius of the circle whose area is 38.5 m^2.***
8. ***The diameter of a circular garden is 28 m. Find the cost of levelling the garden at the rate of Rs. 10 per square metre.***
9. ***Which has the greater area, a square of side 22 cm or a circle of radius 14cm and by how much?***
10. ***The dimensions of a rectangular sheet of cardboard are 21 cm and 7cm. From it a circle of diameter 7 cm has been cut off. Find the area of the remaining sheet.***

3.2.6 (c) Area and Perimeter of sectors of a circle :

Arc :

A part of the circumference of a circle is called an arc. Mark two points A and B on the circle. AB is called an arc of the circle and it is denoted by $\overset{\frown}{AB}$. However in the fig.3.19, if we proceed from A to B in the clockwise direction the arc is a minor arc. If we proceed from A to B in the anticlockwise direction the arc AB is a major arc.

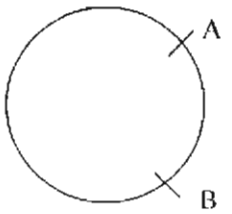


Fig. 3.19

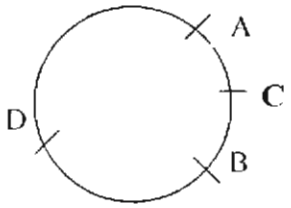


*Fig.*3.20

To be more specific it is necessary to mark the arc by three letters. In fig.3.20, the arc ACB is the minor arc and the arc ADB is the major arc.

Sector:

The area bounded by two radii and an arc cut off by the radii is called a sector. In the fig.3.21 OAB is a sector. $\angle AOB$ is called the angle of the sector OAB.

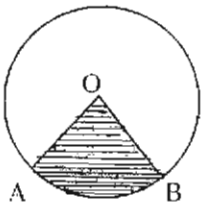


Fig. 3.21

Semicircle:

The diameter of a circle divides the circle into two halves. Each half is called a semicircle. [see fig 3.21(a)]

The semicircle is a sector of a circle whose angle at the centre is 180° .

*Fig.*3.21*(a)*

Quadrant:

Draw two perpendicular diameters to a circle. They divide the circle into four equal parts. Each part is called a quadrant of the circle. [see fig 3.21(b)]. **The quadrant is a sector of a circle whose angle at the centre is 90°.**

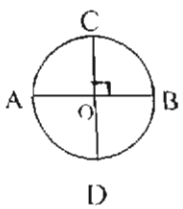


Fig.3.21(b)

Area of a semicircle :

$$\text{Area of a semicircle} = \frac{1}{2} \times (\text{Area of the circle})$$

$$= \frac{1}{2} \times \pi r^2 = \frac{\pi r^2}{2}$$

$$\therefore \textbf{ Area of a semicircle } = \frac{\pi r^2}{2} \textbf{ sq. units}$$

Perimeter of a semicircle:

Perimeter of a semicircle

$= \frac{1}{2}$ (Perimeter of the circle) + diameter of the circle [see fig.3.22]

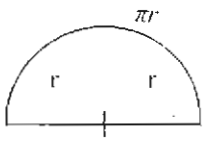


*Fig.*3.22

$$= \frac{1}{2} \times 2\pi r + d$$

$$= \pi r + 2r \text{ (since } d = 2r)$$

$$= (\pi + 2) r$$

$\therefore$ **Perimeter of a semicircle = $(\pi + 2)\, r$ units.**

Area of a quadrant :

$$\text{Area of a quadrant} = \frac{1}{4} \times \text{Area of the circle}$$

$$= \frac{1}{4} \times \pi r^2 = \frac{\pi r^2}{4}$$

$$\therefore \textbf{ Area of a quadrant } = \frac{\pi r^2}{4} \textbf{ sq. units}$$

Perimeter of a quadrant:

Perimeter of a quadrant = $\frac{1}{4} \times$ Perimeter of the circle + 2 r (See fig. 3.23)

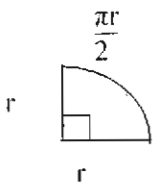


*Fig.*3.23

$$= \frac{1}{4} \times 2\pi r + 2r = \frac{\pi r}{2} = 2r$$

$$= (\frac{\pi}{2} + 2)\, r \text{ units}$$

$\therefore$ **Perimeter of a quadrant = $(\frac{\pi}{2} + 2)$ r units**

Example 29:

The radius of a semicircle is 21 cm. Find the area and perimeter of the semicircle.

Solution:

Here radius , $r = 21$ cm

$$\text{Area of a semicircle} = \frac{\pi r^2}{2} \text{ sq. units}$$

$$= \frac{22}{7} \times \frac{1}{2} \times 21 \times 21 = 693$$

$\therefore$ Area of the semicircle $= 693 \text{ cm}^2$

Perimeter of a semicircle $= (\pi + 2)\, r$ units

$$= \left(\frac{22}{7} + 2\right) \times 21$$

$$= \left(\frac{22+14}{7}\right) \times 21$$

$$= \frac{36}{7} \times 21 = 108$$

$\therefore$ Perimeter of the semicircle $= 108$ cm

Example 30 :

The radius of a circle is 28 cm. Find the area and the perimeter of a quadrant of the circle.

Solution :

Here radius, $r = 28$ cm.

Area of a quadrant $= \frac{\pi r^2}{4}$ sq. units

$$= \frac{22}{7} \times \frac{1}{4} \times 28 \times 28$$

$$= 616$$

$\therefore$ Area of the quadrant $= 616 \text{ cm}^2$

Perimeter of a quadrant $= \left(\frac{\pi}{2} + 2\right) r$ units

$$= \left(\frac{22}{7 \times 2} + 2\right) \times 28$$

$$= \left(\frac{11}{7} + 2\right) \times 28$$

$$= \left(\frac{11+14}{7}\right) \times 28$$

$$= \frac{25}{7} \times 28 = 100$$

$\therefore$ Perimeter of the quadrant $= 100$ cm

Example 31:

Find the area of the shaded region. (Fig 3.24)

Solution :

Area of the shaded region in the fig 3.24

$= 2 \times$ Area of the quadrant

$$= 2 \times \frac{\pi r^2}{4}$$

$$= 2 \times \frac{22}{7} \times \frac{1}{4} \times 7 \times 7 = 77$$

$\therefore$ Area of the shaded region $= 77 \text{ cm}^2$

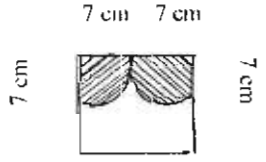


*Fig.*3.24

Exercise 3.10

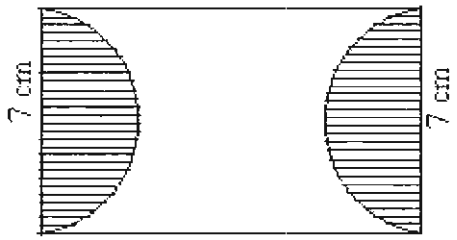
1. Find the areas and perimeters of semicircles whose radii are

(a) 7 cm *(b) 10.5 cm* *(c) 3.5 cm*

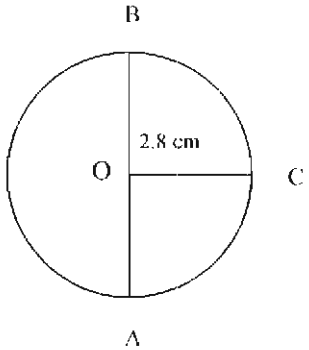
2. Find the areas and perimeters of quadrants whose radii are

(a) 42 cm *(b) 14 cm* *(c) 5.6 cm*

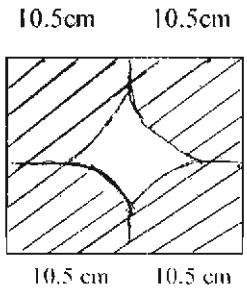
3. Find the area of the shaded region in the figure.



4. Find the area of the semicircle ACB and the quadrant BOC in the figure.



5. Find the area of the shaded region in the figure.



3.2.7 Area of Pathways :

3.2.7 (a) Rectangular Pathway :

Uniform pathway outside a rectangle :

Look at the fig. 3.25. A rectangle is drawn with length '*l*' and breadth '*b*'. Outside the rectangle a uniform path of width '*w*' is drawn. The area of the path is shown by shaded lines.

Length of the inner rectangle = l units
Breadth of the inner rectangle = b units
$\therefore$ Length of the outer rectangle = (l +2w) units
Breadth of the outer rectangle = (b + 2w) units

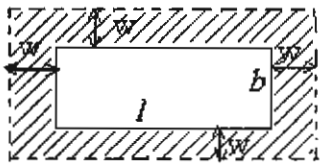


*Fig.*3.25

$\therefore$ Area of the path = [Area of the outer rectangle – Area of the inner rectangle] sq.units

Example 32 :

The length of a building is 10 m and its breadth is 8 m. A path of width 1 m is made all around the building outside. Find the area of the path.

Solution :

Length of the rectangle , l = 10 m
Breadth of the rectangle , b = 8 m
$\therefore$ Area of the inner rectangle = $l \times b$ sq. units
= 10×8
= 80 m^2
Width of the path = 1 m
Length of the outer rectangle = 10 + 2 = 12 m
Breadth of the outer rectangle = 8 + 2 = 10 m
$\therefore$ Area of the outer rectangle = $l \times b$ sq. units
= $12 \times 10 = 120$ m^2

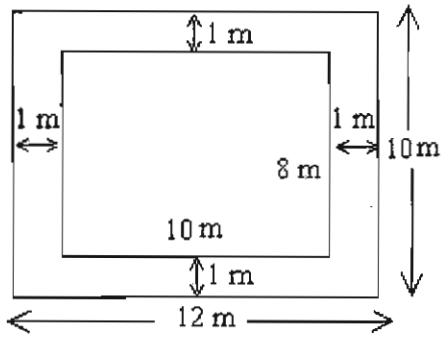


*Fig.*3.26

Area of the path = Area of the outer rectangle – Area of the inner rectangle
= 120 – 80 = 40
$\therefore$ Area of the path = 40 m^2

Uniform path inside a rectangle:

Look at the fig 3.27. A rectangle is drawn with length 'l ' and breadth 'b'. Inside this rectangle a uniform path of width 'w' is drawn. The area of the path is shown by shaded lines.

Length of the outer rectangle = l units
Breadth of the outer rectangle = b units
Width of the path = w units
Length of the inner rectangle = ($l - 2w$) units
Breadth of the inner rectangle = ($b - 2w$) units

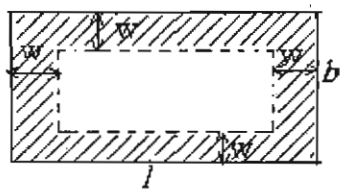


Fig. 3.27

$\therefore$ Area of the path = [Area of the outer rectangle – Area of the inner rectangle] sq.units

Example 33:

The length of a rectangular park is 100 m and its breadth is 75 m. There is a path all around inside the park with uniform width 2 m. Find the cost of levelling the path at the rate of Rs 5 per square metre.

Solution:

Length of the outer rectangle , l = 100 m
Breadth of the outer rectangle, b = 75 m
∴ Area of the outer rectangle = $l \times b$ sq. units
= 100 × 75
= 7500 m^2

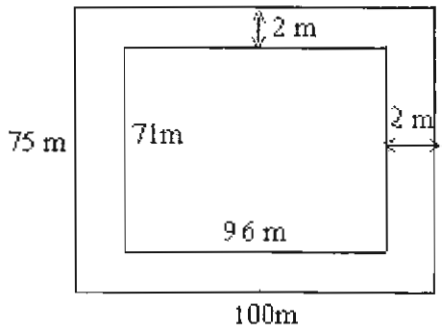


*Fig.*3.28

Width of the path, w = 2 m
Length of the inner rectangle = 100 – 4 = 96 m
Breadth of the inner rectangle = 75 – 4 = 71 m
∴ Area of the inner rectangle = $l \times b$ sq. units
= 96 × 71
= 6816 m^2

Area of the path = [Area of the outer rectangle – Area of the inner rectangle] sq.units
= 7500 – 6816
= 684 m^2

The cost of levelling the path of area 1 m^2 = Rs. 5
The cost of levelling the path of area 684 m^2 = 684 × 5
= 3420
∴ The cost of levelling the path = Rs. 3.420

3.2.7 (b) Circular Pathway :

Concentric Circles:

Circles drawn in a plane with a common centre and different radii are called concentric circles. (Fig. 3.29)

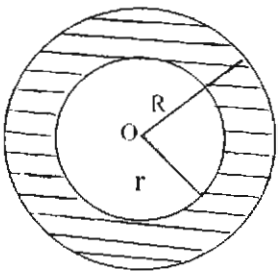


Fig. 3.29

Circular Pathway:

In fig.3.29 , O is the common centre of the two circles.The radius of the outer circle is denoted by R. The radius of the inner circle is denoted by r. The shaded portion is known as the **circular ring** or the **circular Pathway.**

Hence a circular ring is the portion bounded between two concentric circles.

Width of the ring:

The difference between the lengths of the outer radius R and the inner radius r is the width of the ring (pathway). It is usually denoted by w.

∴ The width of the ring, w = [The radius of the outer circle – The radius of the inner circle] units
= (R – r) units

Note: We have seen R – r = w

From this we can get , **(a) R = (w + r) units**

(b) r = (R – w) units

Example 34 :

Find the width of a circular path with outer radius 32 cm and inner radius 24 cm

Solution:

Outer radius , R = 32 cm

Inner radius , r = 24 cm

Width of the circular path , w = R – r

= 32 – 24

= 8

$\therefore$ The width of the circular path = 8 cm

Area of a circular path:

In fig 3.29 , the shaded portion is the area of the circular path.

We notice that ,

The area of the circular path = (Area of the outer circle) – (Area of the inner circle)

$= \pi R^2 - \pi r^2$

$= \pi (R^2 - r^2)$ sq. units.

$\therefore$ **Area of a circular path** $= \pi (R^2 - r^2)$ **sq. units**

$= \pi (R + r)(R - r)$ **sq. units**

Example 35:

Calculate the area of a circular ring whose outer radius is 21cm and its inner radius is 14 cm.

Method 1 :

Outer radius , R = 21 cm

Area of the outer circle = πR^2 sq. units

$= \frac{22}{7} \times 21 \times 21$

$= 1386 \text{ cm}^2$

Inner radius , r = 14 cm

Area of the inner circle = πr^2 sq. units

$= \frac{22}{7} \times 14 \times 14$

$= 616 \text{ cm}^2$

Area of the circular ring = (Area of the outer circle) – (Area of the inner circle)

= 1386 – 616 = 770

$\therefore$ Area of the circular ring = 770 cm^2

Method 2 :

Outer radius , R = 21 cm

Inner radius , r = 14 cm

Area of the circular ring = $\pi (R + r)(R - r)$ sq. units

$= \frac{22}{7} (21 + 14)(21 - 14)$

$$= \frac{22}{7} \times 35 \times 7$$

$$= 770$$

$\therefore$ Area of the circular ring = 770 cm^2

Example 36 :

A play ground is in the form of a circle of radius 70 m. A uniform circular track of width 7 m is laid inside the playground. Find the area of the track.

Solution :

Radius of the outer circle , R = 70 m

$\therefore$ Area of the outer circle = πR^2 sq. units

$$= \frac{22}{7} \times 70 \times 70 = 15400 \text{ m}^2$$

Width of the track , w = 7 m

Radius of the inner circle , r = 70 – 7 = 63 m

$\therefore$ Area of the inner circle = πr^2 sq. units

$$= \frac{22}{7} \times 63 \times 63$$

$$= 12474 \text{ m}^2$$

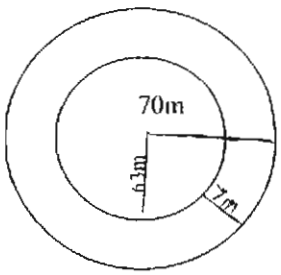


Fig. 3.30

$\therefore$ Area of the track = (Area of the outer circle) – (Area of the inner circle)

= 15400 – 12474 = 2926

$\therefore$ Area of the track = 2926 m^2

Discuss : Find the area of the track using the formula Area, A = π (R + r)(R – r) sq. units

Example 37 :

A uniform circular path of width 3.5 m is laid outside a circular park of radius 56 m. Find the cost of levelling the path at the rate of Rs 3 per m^2.

Solution:

Radius of the inner circle , r = 56m

Width of the path , w = 3.5 m

Radius of the outer circle , R = 56 + 3. 5 = 59.5 m

Area of the path = π (R + r)(R – r) sq. units

$$= \frac{22}{7}(59.5 + 56)(59.5 - 56)$$

$$= \frac{22}{7}(115.5)(3.5)$$

$$= 1270.50 \text{ m}^2$$

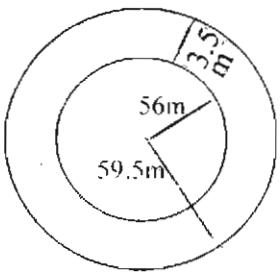


Fig. 3.31

The cost of levelling the path of area 1 m^2 = Rs 3

The cost of levelling the path of area 1270.50 m^2 = 1270.50 × 3

= 3811.50

$\therefore$ The cost of levelling the path = Rs 3811.50

Exercise 3.11

1. *The length and breadth of a rectangular garden are 30 m and 20 m respectively . A path of uniform width 3m is laid outside. What is the area of the path?*
2. *The length and breadth of a playground are 150 m and 100 m respectively. A road of width 5m is laid outside the playground. Find the cost of levelling the road at the rate of Rs. 4 per m^2.*
3. *A black board is 4m long and 2m broad. A frame of width 0.2m is fixed outside the blackboard. If the cost of the frame is Rs 5 per m^2 , find the total cost of the frame.*
4. *The length and breadth of a room are 5 m and 4 m respectively. A green colour border of uniform width of 0.25 m has been made around inside the room. Find the area of the border.*
5. *The length and breadth of a door are 2 m and 1 m respectively. The door has a border of width 0.1 m around inside the door. Find the area of the border.*
6. *The outer radius and the inner radius of some circular rings are given below. Find the width of each ring.*
   (a) R = 15 cm, r = 8 cm (b) R = 10.5 m, r = 7 m (c) R = 28 dm, r = 23 dm
7. *The radius of a circular garden is 49 m. A circular path of width 7 m is laid outside the garden. Find the area of the path.*
8. *The diameter of a circular park is 70 m. Outside the park there is a circular path of 3.5 m width. Find the cost of levelling the path at the rate of Rs 3 per sq m..*
9. *A circular field of radius 35 m has a circular path of width 3.5 m around inside its boundary. Find the area of the path.*
10. *A circular carpet of diameter 4.2 m is spread in a room of length 6m and breadth 5m. Find the area of the floor not covered by the carpet.*

3.3 Volumes :

Volume:

The space occupied by any object is called the volume of that object.

Unit of Volume :

Look at the fig.3.32. Its each edge is 1cm long.
It is a cube of side 1cm.Hence we call it as **1 centimetre cube.** The amount of space occupied by this cube is **1 cubic centimetre**. We simply write it as **1cu.cm** or **1 cm^3** . We call this cube as a **unit cube.** It is the standard unit for measuring volumes.

The Cubic centimetre or cm^3 is used to express the volume of solids.

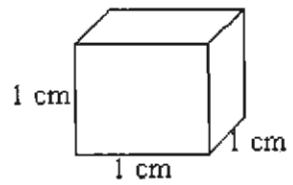


Fig. 3.32

Note: If a cube of side 1m long is used to measure the volume of solids, then the unit of volume is cubic metre or simply m^3.

3.3.1 Cuboid
3.3.2 Cube

3.3.1 Cuboid :

We have seen most of the things sold in the market are packed in tins, cardboard boxes, thick paper boxes etc. For example chalk pieces , Television , Computer and match sticks are packed in boxes.
These boxes have the shape of a Cuboid.

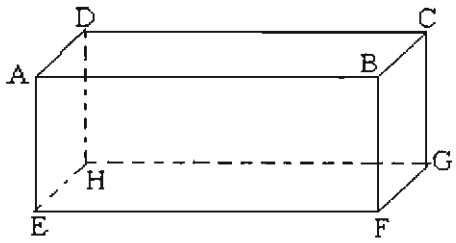


Fig. 3.33

The surface of a cuboid consists of six rectangular **Faces.** Any two faces other than the opposite faces are called adjacent faces. The adjacent faces meet along a line segment. This line segment is called an **Edge** of the Cuboid. There are 12 edges in a Cuboid. The point of intersection of three edges of a cuboid is called a **Vertex** of the Cuboid. A Cuboid consists of eight vertices.

In the fig.3.33, ABCDEFGH is a cuboid .

(a) ABCD , EFGH , EFBA , HGCD , EHDA and FGCB are the six faces of the cuboid.

(b) AB , CD , EF , GH , AE , DH , BF , CG , AD , EH , BC and FG are the twelve edges.

(c) A , B , C , D , E , F , G and H are the eight vertices.

Taking in particular the rectangular face EFGH , EF is the length (l), FG is the breadth (b) and FB is the height (h) of the cuboid.

3.3.1 (a) To find the volume of a cuboid :

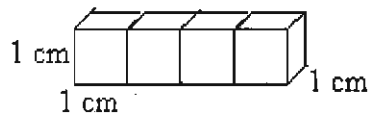


Fig. 3.34

The number of unit cubes contained in a cuboids is its volume.

Take four cubes of same size, each 1cm cube. Arrange them in a row as shown in fig.3.34. What is the shape of the solid formed? It is a cuboid, isn't it? It contains 4 one-centimetre cubes. Hence the volume of this cuboid is 4 cm^3. We notice that its length is 4 cm, breadth is 1 cm and height is 1cm.

$\therefore$ Volume of the cuboid =4 cm × 1cm × 1cm = 4 cm^3.

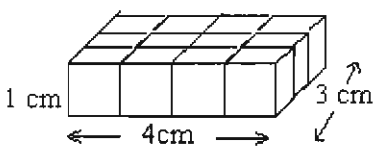


Fig. 3.35

Take 12 cubes of same size , each 1cm cube. Arrange them in 3 rows as shown in fig.3.35. The shape we obtained is also a cuboid. Isn't it ? It contains 12 one-centimetre cubes. Hence the volume of this cuboid is 12 cm^3. Its length is 4cm. breadth is 3 cm and height is 1 cm.

$\therefore$ Volume of the cuboid = 4 cm × 3 cm × 1 cm = 12 cm^3

Now take 24 one-centimetre cubes and arrange them as shown in fig.3.36.

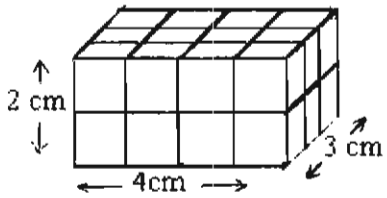


Fig. 3.36

Again we obtain a cuboid. This cuboid contains 24 one-centimetre cubes. Hence the volume of this cuboid is 24 cm^3.

It's length is 4 cm , breadth is 3 cm and height is 2 cm.

$\therefore$ Volume of the cuboid = 4 cm × 3 cm × 2 cm = 24 cm^3.

Now look at the following table prepared from the shapes discussed above:

Cuboid	Length (l) (in cm)	Breadth (b) (in cm)	Height (h) (in cm)	Total number of 1 cm cubes	Volume ($l \times b \times h$) (in cm^3)
1	4	1	1	4 ($4 \times 1 \times 1$)	4
2	4	3	1	12 ($4 \times 3 \times 1$)	12
3	4	3	2	24 ($4 \times 3 \times 2$)	24

Table 3.2

From the table 3.2 we observe that

The volume of a cuboid = (length × breadth × height) cu.units

(i.e) V = $l \times b \times h$ cu. units = lbh cu.units , where V is the volume of the cuboid and l,b and h are the length, the breadth and the height of the cuboid respectively.

Note 1: Length , breadth and height of the cuboid must be expressed in the same units.

2. From the above formula, we also obtain that

(a) length , $l = \frac{V}{b \times h}$ units

(b) breadth , $b = \frac{V}{l \times h}$ units

(c) height , $h = \frac{V}{l \times b}$ units

Example 38 :

Find the volume of a cuboid of length 6m, breadth 4m and height 3m .

Solution :

Here $l = 6$ m, $b = 4$ m, $h = 3$ m

Volume of the cuboid , V $= l \times b \times h$ cubic units

$= 6 \times 4 \times 3$

$= 72$

$\therefore$ Volume of the cuboid $= 72 \text{ m}^3$

Example 39 :

How many bricks each of length 30 cm , breadth 15 cm and height 6 cm will be needed to construct a wall of length 15 m , height 5 m and thickness 25.2 cm ?

Solution :

Length of the brick, $l = 30$ cm

Thickness of the brick, $b = 15$ cm

Height of the brick, $h = 6$ cm

$\therefore$ The volume of one brick, V $= l \times b \times h$ cu. units

$= 30 \times 15 \times 6 \text{ cm}^3$

Length of the wall, $l = 15$ m $= 1500$cm

Breadth of the wall, $b = 25.2$ cm

Height of the wall, $h = 5$ m $= 500$cm

$\therefore$ Volume of the wall, V $= l \times b \times h$ cu. units

$= 1500 \times 25.2 \times 500 \text{ cm}^3$

$$\text{Number of bricks required} = \frac{\text{Volume of the wall}}{\text{Volume of 1 brick}}$$

$$= \frac{1500 \times 25.2 \times 500}{30 \times 15 \times 6}$$

$$= 7000$$

$\therefore$ The number of bricks required to construct the wall = 7000

Example 40:

A cuboid vessel is 10 cm long and 8 cm broad. How high it must be made to hold 480 cm^3 of liquid ?

Solution:

Here V $= 480 \text{ cm}^3$, $l = 10$ cm , $b = 8$ cm

$$h = \frac{V}{l \times b} \text{ units}$$

$$= \frac{480}{10 \times 8} = 6$$

$\therefore$ The height of the vessel $= 6$ cm

3.3.2 Cube :

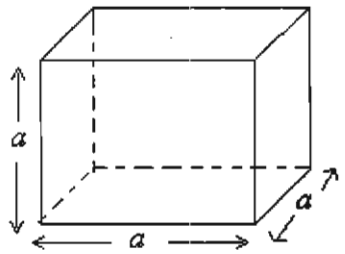


Fig. 3.37

Cube is a special type of cuboid whose length , breadth and height are all equal. (Fig. 3.37)

That is each side (or) edge of a cube is 'a' units .

The volume of the cube is given by

$$V = (\text{Length} \times \text{breadth} \times \text{height}) \text{ cu. units}$$
$$= (\text{Side} \times \text{Side} \times \text{Side}) \text{ cu. units}$$

That is $V = a \times a \times a$

$$V = a^3 \textbf{ cu. units}$$

where V is the volume of the cube and 'a' is the side (or edge)of the cube.

Example 41:

Find the volume of a cube whose edge measures 5 cm.

Solution :

Here a = 5 cm

$\therefore$ Volume of the cube , $V = a^3$ cu.units

$= 5 \times 5 \times 5$

$= 125$

$\therefore$ Volume of the cube = 125 cm^3

Exercise 3.12

1. Draw a diagram to represent a cube. Label its vertices and then write the names of its

(a) faces *(b) edges* *(c) vertices.*

2. Find the volumes of the cuboids with the following measurements.

	Length	*Breadth*	*Height*
(a)	*5 cm*	*3 cm*	*1 cm*
(b)	*14 cm*	*10 cm*	*1.5 cm*
(c)	*10 m*	*6 m*	*2 m*
(d)	*2.5 dm*	*11 cm*	*23 cm*
(e)	*6 m*	*1.5 m*	*2 m*

3. The length, breadth, height and volume of the cuboids are given below. Fill in the dimensions left out , by calculation .

	Length	*Breadth*	*Height*	*Volume*
(a)	*10 cm*	_____	*2 cm*	*60 cm^3*
(b)	*100 cm*	*50 cm*	_____	*125000 cm^3*
(c)	*24 m*	_____	*8 m*	*2880 m^3*
(d)	_____	*15 cm*	*12 cm*	*3600 cm^3*

4. A metal box has dimensions 4 cm , 2.5 cm and 1.5 cm. Find the volume of the packet which can hold 20 such boxes.

5. A platform of the following dimensions is to be constructed, with length 10 m, breadth 2.25 m and height 60 cm. How many bricks of dimensions 25 cm × 12 cm × 9 cm are required to construct the platform ?

6. A rectangular overhead watertank has its internal length, breadth and height equal to 5 m , 3 m , 1.5 m respectively. How many litres of water can it hold ?

(1 cu. m =1000 litres)

7. Find the volumes of the cubes whose sides are (a) 8 cm (b) 3.5 m (c) 25 mm

THINGS TO REMEMBER

1. The time tables worked out by the railway or the Interstate bus service are planned according to the 24 hour clock .

2. Area of a triangle = $\frac{1}{2}$ *bh* sq.units.

3. Area of a parallelogram = *bh* sq. units.

4. Area of a rhombus = $\frac{1}{2}$ $d_1 d_2$ sq.units.

5. Area of a trapezium= $\frac{1}{2}$ $(a + b) \times h$ sq.units.

6. Area of a circle = πr^2 sq.units.

7. Area of a semi circle = $\frac{\pi r^2}{2}$ sq.units.

8.Area of a quadrant = $\frac{\pi r^2}{4}$ sq.units.

9.The circumference (Perimeter) of a circle = πd (or) $2\pi r$ units.

10. Perimeter of a semicircle = $(\pi + 2)\, r$ units.

11. Perimeter of a quadrant = $(\frac{\pi}{2} + 2)\, r$ units.

12. Area of a rectangular path = (Area of outer rectangle – Area of inner rectangle) sq.units.

13. Area of a circular ring = (Area of outer circle – Area of inner circle) sq.units.

$= \pi(R^2 - r^2)$ sq.units

$= \pi(R + r)(R - r)$ sq.units

14. Width of a circular ring , w = *(R –r)* units.

15. Volume of a cuboid , V = *lbh* cu .units.

16. Volume of a cube , V = a^3 cu. Units.

UNIT TEST – 3

Part-I *(10 × 1 = 10)*

Choose the correct answer :

1. Area of a rectangle is

(a) bh sq. units *(b) lb sq. units* *(c)* $\frac{1}{2}$ *bh sq. units* *(d)* $\frac{1}{2} \times d_1 \times d_2$ *sq. units*

2. 1 Are is equal to _____

(a) $100\ m^2$ *(b)* $10000\ m^2$ *(c)* $1000\ m^2$ *(d)* $100\ cm^2$

3. Radius of a circle is 7cm. Its diameter is

(a) 49 cm *(b) 3.5 cm* *(c) 14 cm* *(d)* $\frac{22}{7}$ *cm*

4. Perimeter of a semicircle is

(a) $(\pi + 2)\, r$ *units* *(b)* $\frac{\pi r^2}{4}$ *sq. units* *(c)* $\frac{\pi r^2}{2}$ *sq.units* *(d)* $2\pi r$ *units*

5. Volume of a cuboid is

(a) a^3 *cu. units* *(b)* a^2 *sq. units* *(c)* $2(l + b)$ *units* *(d) lbh cu.units*

Fill in the blanks:

6. The number of faces of a cuboid is _____

7. 13.15 hrs = _____ p.m

8. Area of a rhombus is _____

9. Outer radius of a circular ring is 12 cm. Its inner radius is 9 cm . Width of the ring is____

10. Area of a quadrant is _____.

Part – II *(10 × 2= 20)*

Answer in one or two steps :

11. Vaigai Express departs from Madurai at 06.45 hrs. It arrives Chennai Egmore at 14.30 hrs. Find the journey time.

12. Find the perimeter of a square whose side is 9 cm.

13. Find the area of a triangle whose base is 12 cm and height is 5 cm.

14. Area of a parallelogram is 360 cm^2. Its height is 15 cm. Find its base.

15. The diagonals of a rhombus are 9 cm and 12 cm. Find its area.

16. Outer radius of a circular ring is 2.5 m and the width of the ring is 50 cm. Find the inner radius of the circular ring.

17. The length , breadth and height of a cuboid are 8 cm, 5 cm and 2 cm. Find its volume.

18. Find the volume of a cube whose side is 6 m.

19. Convert (a) 19.40 hrs into ordinary time (b) 7.25 a.m into railway time.

20. Convert (a) 12500 m^2 into ares. (b) 2.5 hectares into sq.meters.

Part – III *(4 × 5 = 20)*

Give detailed answer :

21.

Route	*Departure from Chennai*	*Arrival at Tirunelveli*	*Fare from Chennai (Rs)*
Nellai Express	*21.00 hrs*	*08.35hrs (next day)*	*223*
SETC Bus	*19.00 hrs*	*08.30 hrs (next day)*	*200*

From the time table given above, find which journey is faster? Find whether the quicker journey is also the cheaper journey .

22. The area of a trapezium is 150 cm^2. Its parallel sides are 15cm and 10cm. Find the distance between the parallel sides.

23. A rectangular park of length 40 m and breadth 25 m is to be surrounded externally by a path which is 2 m wide. Calculate the cost of levelling the path at the rate of Rs.2 per sq. m.

24. How many bricks will be required to construct a wall of length 10 m , height 3 m and thickness 27 cm, if each brick measures 25 cm x 12 cm x 9 cm?

4. ALGEBRA

Algebra is a generalised form of Arithmetic. In Arithmetic we use numerals to represent numbers. But in Algebra we use literals along with numerals to represent numbers. We have already learnt some basic ideas of Algebra. In this chapter we shall learn more about it.

4.1 Revision
4.2 Polynomial expression
4.3 Operations on polynomials
4.4 Finding the value of an expression
4.5 Equations

4.1 Revision :

Basic ideas are essential for further study of Algebra. Hence let us revise the following basic ideas.

Literals :

The letters that are used to represent numbers are known as **literals.**

For example consider the statement The sum of two numbers is 18 '. It can be written as $x + y = 18$. Here x and y are literals.

Constant:

A term or a quantity which takes a fixed numerical value is called a **constant.**

For example , 6 , – 8 , $\frac{3}{7}$, 2.5 are constants.

Variable :

A term or a quantity which takes different numerical values is called a **variable.**

Variables can be represented by any alphabet such as p , q , r , x , y , z etc.

Note: Consider the term $7x$. Here 7 is a constant and x is a variable but $7x$ is a variable.

It is because as the value of x changes, the value of $7x$ also changes. Hence the combination of a constant and a variable is always a variable.

Power or Exponent or Index of a variable :

The **Power** of a variable is the number of times the variable has been multiplied by itself. Here we call the variable as the base.

For example, $x \times x \times x \times x \times x = x^5$. In x^5, the variable x is the base and 5 is the power.

Note: Consider the variable y. Here y is the base and the power of y is 1.

$\therefore y^1 = y$

Numerical Coefficient :

The numerical part of a variable is the coefficient of the variable.

For example, In $9a$, 9 is the coefficient of ' a '.

In $-15xy$, -15 is the coefficient of xy.

In $-m$, -1 is the coefficient of m.

Note: When the coefficient of a variable is one , we write only the variable.

For example $1 \times m$ is written as m. Here the coefficient of m is 1.

Terms :

A constant or a variable or a combination of constant and variables combined by means of multiplication or division is called a term.

For example, (a) 18 is a term which is a constant.

(b) x is a term which is a variable.

(c) $-10m$ is a term which is the product of a constant and a variable.

(d) $\frac{p}{12}$ is a term which is the quotient of a variable and a constant.

Like terms and unlike terms :

Terms having the same variable or variables are called **like terms.**

For example, (a) $3x$, $-11x$, $\frac{x}{8}$ are like terms since x is the variable in all the terms.

(b) $a^2 b$, $6a^2 b$, $-15a^2 b$ are like terms since $a^2 b$ is the variable in all the terms.

Note: The like terms may differ only in their numerical coefficient.

Terms having different variables are called **unlike terms.**

For example, (a) $4x$, $-6y$, $\frac{z}{14}$ are unlike terms since the variables are different.

(b) $x\,y$, $-12x^2y$, $5xy^2$ are unlike terms since the powers of variables are different.

(c) $5x$, $5y$, $5z$ are unlike terms since the variables are different.

Note: In an algebraic term if the coefficient of a variable is changed, we get another like term.

For example in the term $5x$ if we change 5 to 7 we get $7x$, which is a like term of $5x$.

If the variable or the power of the same variable is changed we get an unlike term.

For example in the term $5x$ if the variable x is changed to y then $5x$ and $5y$ are unlike terms.

[Similarly $5x$ and $5x^2$ are unlike terms.]

Addition and Subtraction of like terms :

We can add or subtract like terms.

For example, (a) Add: $5x^3$, $9x^3$ and x^3

$$5x^3 + 9x^3 + x^3 = (5 + 9 + 1)\,x^3 \text{ (add the coefficients)}$$
$$= 15x^3$$

(b) Simplify: $12a^2 b^3 - 7a^2 b^3$

$$12\,a^2 b^3 - 7\,a^2 b^3 = (12 - 7)\,a^2 b^3$$
$$= 5a^2 b^3$$

Exercise 4.1

1. ***Write the following statements using literals.***
   (a) Five times of a number is 15.
   (b) Which number multiplied by itself gives 36?
   (c) What number is to be added with 10 to get 22?

2. *Identify the constants and variables from the following:*
   $15, a, x^2, -8, 7.5, m^3, p, -q$
3. *Write the base and power of the following terms:*
   (a) a^3 (b) m^2 (c) q (d) y^5 (e) x^4
4. *Write the numerical coefficient of the following terms:*
   (a) $5x^2$ (b) $-7xy$ (c) $-\frac{3}{8}a^2b$ (d) p (e) $-27m^3$
5. *Identify the like terms and unlike terms:*
   (a) $3x$ and $5x$ (b) $7x$ and $-5y$ (c) $3a^2b$ and $-6a^2b$ (d) $-8y^3$ and $3y^3$
   (e) $10p^2q$ and $8pq^2$
6. *Find the sum of the following terms:*
   (a) $5x$ and $7x$ (b) $7x^2y$ and $-6x^2y$ (c) $2m$ and $11m$
   (d) $18ab$, $5ab$ and $-12ab$ (e) $3x^2$, $-7x^2$ and $8x^2$
7. *Subtract the first term from the second term.*
   (a) $2xy$, $7xy$ (b) $5a^2$, $10a^2$ (c) $12y$, $3y$ (d) abc, $4abc$
   (e) $6x^2y$, $4x^2y$

4.2 Polynomial expression :

4.2.1	**Arithmetic expression**
4.2.2	**Algebraic expression**
4.2.3	**Polynomial expression**
4.2.4	**Types of polynomials**
4.2.5	**Degree of a polynomial in one variable.**
4.2.6	**Standard form of a polynomial.**

4.2.1 Arithmetic expression:

A number or combination of numbers formed using the arithmetic operations is called a numerical expression or an arithmetic expression.

For example, 7×5, $9 + 25 - 18$, $6 \times 8 \div 5$ are arithmetic expressions.

4.2.2 Algebraic expression :

A combination of numerals, variables and arithmetical operations such as $+, -, \times$ and $\div$ is known as an algebraic expression.

$x + y$, xy, $a^2 - 2b + \frac{c}{5}$, $x^2 - y^2$, $7xy + \frac{2a}{c} - 3m + 5$ are some of the examples of algebraic expressions.

In an algebraic expression we have one or more terms.

For example, $2x^2$, $-5x$, 7 are the terms of the expression $2x^2 - 5x + 7$. In this algebraic expression we have 3 terms.

Example 1 :

Classify into arithmetic expressions and algebraic expressions:

(a) $a + b - c + 9$ (b) $256 \div 2 \times 3$ (c) $x + y + z + 5$ (d) $x^3 - 4x^2 + 5x + 10$
(e) $9 + 18 - 3$

Solution :

The arithmetic expressions are (b) $256 \div 2 \times 3$ and (e) $9 + 18 - 3$.
The algebraic expressions are (a) $a + b - c + 9$, (c) $x + y + z + 5$
and (d) $x^3 - 4x^2 + 5x + 10$.

Example 2 :

Find the number of terms in each of the following algebraic expressions:
(a) $5x - 7$ (b) $6x^2 - 3xy + 7y^2 + 9$ (c) $-2abc$ (d) $a^2 - 2ab + b^2$
(e) $36mn - 8$

Solution :

(a) $5x - 7$ has 2 terms.
(b) $6x^2 - 3xy + 7y^2 + 9$ has 4 terms.
(c) $-2abc$ has only one term.
(d) $a^2 - 2ab + b^2$ has 3 terms.
(e) $36mn - 8$ has 2 terms.

4.2.3 Polynomial expression :

Consider the algebraic expression $6x^3 - 8x^2 + 3$. It has 3 terms, namely $6x^3$, $-8x^2$ and 3.
The power of x in the term $6x^3$ is 3.
The power of x in the term $-8x^2$ is 2.
The third term 3 is a constant term or an independent term.
We note that in this expression the powers of the variable x are whole numbers.
Hence we call this algebraic expression a **Polynomial Expression.**

Discuss:

In the above expression can you tell the power of x in the term 3?
Does this term contain the variable x?
No, it is only a constant. (i.e), it is an independent term.
But we can write 3 as $3 \times x^0$. [Since the value of x^0 is 1]
Hence the power x in the term 3 is 0.

Note: The power of any variable in the independent term is always 0.

Now consider another algebraic expression $2x^3 - 3x^{3/2} + 7x + 10$.
It has 4 terms namely $2x^3$, $-3x^{3/2}$, $7x$ and 10.
In the second term $-3x^{3/2}$, the power of x is $\frac{3}{2}$. The power is not a whole number. Therefore this algebraic expression cannot be called as a polynomial expression.
Therefore **In an algebraic expression if the power of each variable is a whole number, it is called a polynomial expression or simply a polynomial.**

4.2.4 Types of polynomials :

Polynomials are named according to the number of terms present in them.

Monomial :

A polynomial expression containing only one term is called a **monomial.**

For example $5x$, $-mn$, $3a^2$ and $\frac{7}{5}y$ are monomials since each expression contains only one term.

Note: In Greek 'mono' means 'single'.

Binomial :

A polynomial expression containing two terms is called a **binomial.**

For example the following expressions $3x + 2y$, $a^2 - b^2$, $m - 5$ and $7x^2 - \frac{5}{6}x$ contain two terms each. Hence they are called as binomials.

Note: In Latin 'bi' means 'double'.

Trinomial :

A polynomial expression containing three terms is called a **trinomial.**

For example $x + y + z$, $a - 3b + 5c$ and $m^3 - 3m - 10$ are all trinomials.

Note: In Latin 'tri' means 'three'.

Expressions having more than three terms are called as **multinomials** or **polynomials**.

For example, (a) $7x^2 - 3xy - 5x + 8$ is a polynomial of four terms.

(b) $a^3 + 2a^2b + 5ab^2 - 6b^3 + 9$ is a polynomial of five terms.

Note: Monomial, binomial and trinomial are only special names given to a polynomial. Hence monomial, binomial and trinomial are also polynomials.

Example 3 :

Which of the following expressions are polynomials?

(a) $x + 1$ (b) $3m^2$ (c) $2x^2 - x - 4$ (d) $a^{2/5}$ (e) $\frac{4}{3}p^2 - 5q$

(f) $x + x^{1/2} + 5$

Solution :

(a) $x + 1$ (b) $3m^2$ (c) $2x^2 - x - 4$ and (e) $\frac{4}{3}p^2 - 5q$ are polynomials.

(d) $a^{2/5}$ is not a polynomial since the power of the variable 'a' is $\frac{2}{5}$ which is not a whole number.

(f) $x + x^{1/2} + 5$ is not a polynomial since in the second term $x^{1/2}$, the power of x is $\frac{1}{2}$ which is not a whole number.

Example 4 :

Classify the following into monomials, binomials and trinomials:

(a) $3x - 4$ (b) $-5m^2$ (c) $\frac{1}{2}x + 4$ (d) $a^2 - 3a + 7$

(e) $x + y$ (f) $10abc$ (g) $xy - z + 9$ (h) $3m^2 - 4n^2 - 18$

(i) $-5a^2b$ (j) $15x^2y^3z$ (k) $x^3 + x$

Solution :

(a) $3x - 4$ has 2 terms. $\therefore$ it is a binomial.

(b) $-5m^2$ has 1 term. $\therefore$ it is a monomial.

(c) $\frac{1}{2}x + 4$ has 2 terms. $\therefore$ it is a binomial

(d) $a^2 - 3a + 7$ has 3 terms. $\therefore$ it is a trinomial.
(e) $x + y$ has 2 terms. $\therefore$ it is a binomial.
(f) $10abc$ has 1 term. $\therefore$ it is a monomial.
(g) $xy - z + 9$ has 3 terms. $\therefore$ it is a trinomial.
(h) $3m^2 - 4n^2 - 18$ has 3 terms. $\therefore$ it is a trinomial.
(i) $-5a^2b$ has 1 term. $\therefore$ it is a monomial.
(j) $15x^2y^3z$ has 1 term. $\therefore$ it is a monomial.
(k) $x^3 + x$ has 2 terms. $\therefore$ it is a binomial.

Example 5 :

Using the terms given below form any (a) two monomials (b) two binomials (c) two trinomials.

$5x^3$, $-7x^2$, $-10x$, 6

Solution :

(a) Monomials :	(i) $5x^3$	(ii) $-10x$
(b) Binomials:	(i) $5x^3 - 7x^2$	(ii) $6 - 10x$
(c) Trinomials :	(i) $5x^3 - 10x + 6$	(ii) $6 - 10x - 7x^2$

Note : Using the given terms we can form some more polynomials also.

4.2.5 Degree of a polynomial in one variable:

Consider the expression $5x^3 - 7x^2 - 2x + 6$. We know that it is a polynomial in x.

The power of x in the term $5x^3$ is 3.

The power of x in the term $-7x^2$ is 2.

The power of x in $-2x$ is 1.

The power of x in 6 is zero since 6 is a constant term. (since $6 = 6 \times 1 = 6 \times x^0$)

The highest power of x in this polynomial is 3. Hence the degree of the polynomial is 3.

Consider the polynomial $15m - 3m^2 + 6$. The highest power of m in this polynomial is 2. Hence the degree of the polynomial is 2.

Hence **The highest power of the variable in a polynomial of one variable is called the degree of the polynomial.**

Example 6 :

Find the degree of the following polynomials:

(a) $m^3 - 6m + 4$ (b) $x^2 - 2x - 5$ (c) $12 + 3y^2 - 8y + 7y^4$

Solution :

(a) In $m^3 - 6m + 4$, the highest power of m is 3.
$\therefore$ The degree of the polynomial is 3.

(b) In $x^2 - 2x - 5$, the highest power of x is 2.
$\therefore$ The degree of the polynomial is 2.

(c) In $12 + 3y^2 - 8y + 7y^4$, the highest power of y is 4.
$\therefore$ The degree of the polynomial is 4.

4.2.6 Standard form of a polynomial :

Consider the polynomial $x^2 - 3x + 6$. It has 3 terms.

The power of x in the first term x^2 is 2.

The power of x in the second term $-3x$ is 1.
The third term 6 is a constant term and the power of x is zero.
This polynomial is in descending order of the powers of x. Hence this polynomial is said to be in the standard form.
Consider the polynomial $5a^2 + 4a^3 + 10a - 5$.
The power of a in the first term is 2.
The power of a in the second term is 3.
The power of a in the third term is 1.
The fourth term -5 is a constant term and the power of a is zero.
The powers of a are not in descending order. Hence this polynomial is not in the standard form.
From the above we can say that
A polynomial written in descending order of the powers of the variable is said to be in the standard form.
Example 7 :
Write the following polynomials in the standard form:

(a) $x^2 - x^3 + x + 2x^4 - 10$ (b) $y^2 - 12y^3 + 7 - 3y$ (c) $a - \frac{3}{5} + \frac{a^2}{2}$

Solution :

(a) $x^2 - x^3 + x + 2x^4 - 10$ is not in the standard form.
The power of x in the first term is 2.
The power of x in the second term is 3.
The power of x in the third term is 1.
The power of x in the fourth term is 4.
The power of x in the fifth term is 0.

Hence rewriting the terms in the descending order of the power of x, the standard form of the given polynomial is $2x^4 - x^3 + x^2 + x - 10$.

(b) The standard form of $y^2 - 12y^3 + 7 - 3y$ is $-12y^3 + y^2 - 3y + 7$

(c) The standard form of $a - \frac{3}{5} + \frac{a^2}{2}$ is $\frac{a^2}{2} + a - \frac{3}{5}$

Exercise 4.2

1. ***Which of the following are algebraic expressions?***
   (a) $x^2 - 5yz$ *(b)* $2 + m + n$ *(c)* $215 \div 5 \times 2$ *(d)* $11a^2 + 6b^2 - 5$
   (e) $310 - 15 + 62$
2. ***Find the number of terms in each of the following expressions:***
   (a) $3x^3 + 7x - 8x^2 + 9$ *(b)* $4ab - cd + 2$ *(c)* $6xyz$ *(d)* $a^3 + b^3$
   (e) $7m^2n - 8mn + 2$ *(f)* $15pqr - 7t$
3. ***Which of the following are polynomials?***
   (a) $x^3 - 8$ *(b)* $2a^2 - 5a - 7b$ *(c)* $3y^2 - 2y^{3/2} + 7$
   (d) $4p^3 - 7p^2 + 6p - 8$
   (e) $-15xy^2z^5$ *(f)* $5m - 3m^{1/2} + 7$
4. ***Classify the following into monomials, binomials and trinomials:***
   (a) $x^2 + 4x$ *(b)* $-5abc$ *(c)* $3x + 5y + 4z$ *(d)* m^2n
   (e) $2p^3 - \frac{3}{4}p^2 - 6$ *(f)* $8n^3 - 5n^2$
5. ***Using the terms given below, form monomials, binomials and trinomials (two each):***
   $3x, -7x^2, 5x^3, -16$

6. ***Write the degree of each of the following polynomials.***
   (a) $x^3 + x^2 - 3$ *(b)* $5m^2 - 6m$ *(c)* $7 - 6y + y^4 - 10y^3$ *(d)* $2z^5 - 11z$
   (e) $1 - 7a$
7. ***Write the given polynomials in the standard form:***
   (a) $5x^3 + 3x + 18 - 4x^2$ *(b)* $8 - 3x^2 + 4x$ *(c)* $-2m + 6 - 3m^2$ *(d)* $y^3 + 1 + y - 3y^2$
   (e) $5 - 3a + 4a^2$

4.3 Operations on polynomials :

We know that a combination of numerals, variables and arithmetical operations such as $+$, $-$, $\times$ and $\div$ is an algebraic expression. Hence the algebraic expressions obey all the rules of addition, subtraction, multiplication and division of numbers. In this chapter we shall study the four basic operations on polynomials.

4.3.1	**Addition of polynomials**
4.3.2	**Subtraction of polynomials**
4.3.3	**Multiplication of monomials**
4.3.4	**Division of monomials**

4.3.1 Addition of polynomials :

We can add two or more polynomials having like terms.

4.3.1 (a) Addition of polynomials having like terms:

We have learnt that the sum of two or more like terms is a like term whose coefficient is the sum of the coefficients of all the like terms.

Example 8 :

Add: $3x$, $8x$ and $14x$

Solution :

$$3x + 8x + 14x = (3 + 8 + 14)\, x = 25x$$

Note: Since all the coefficients are positive, the coefficients are added.

Example 9 :

Add: $-5\,mn$, $-\,mn$ and $-\,6mn$

Solution :

$(-5mn) + (-mn) + (-6mn) = [(-5) + (-1) + (-6)]\, mn = -12mn$

Note: Since all the coefficients are negative, the coefficients are added without considering the signs and then the negative sign is put.

When all the terms are not of same sign,

(a) first add all the coefficients with positive sign.
(b) next add all the coefficients with negative sign.
(c) then without considering their sign subtract the smaller coefficient from the greater coefficient and put the sign of the greater coefficient.

Example 10 :

Add : $4x$, $8x$, $-\,2x$ and $-\,5x$

Solution :

$$4x + 8x + (-2x) + (-5x)$$

Here the terms $4x$ and $8x$ have positive coefficients.
$\therefore$ Adding we get $4x + 8x = 12x$
The terms $-2x$ and $-5x$ have negative coefficients.
$\therefore$ Adding we get $-2x - 5x = -7x$
In the terms $12x$ and $-7x$, 12 is greater than 7.
Hence $12x - 7x = 5x$
$\therefore$ The sum is $5x$

To find the sum of polynomials , we need to add their like terms. For this, arrange each of the polynomials one below the other in the order of their like terms.

Example 11 :

Add: $x + 2y + 3z$ and $4x - y + z$

Solution :

$$\begin{array}{r} x + 2y + 3z \\ 4x - y + z \\ \hline 5x + y + 4z \end{array}$$

$\therefore$ The sum is $5x + y + 4z$

Example 12 :

Add : $5a + 3b - 4c$; $2a + 6b$ and $a - 4b + 3c$

Solution :

$$\begin{array}{r} 5a + 3b - 4c \\ 2a + 6b \\ a - 4b + 3c \\ \hline 8a + 5b - c \end{array}$$

$\therefore$ The sum is $8a + 5b - c$

Example 13 :

Add: $6 + 7x^3 - 2x^2 - 5x$; $3x^2 - 9 + 6x$ and $x^3 - 4x + 2$

Solution :

(**Note:** Write the polynomials in the standard form before adding them)

Here $6 + 7x^3 - 2x^2 - 5x$ and $3x^2 - 9 + 6x$ are not in the standard form..

$\therefore$ Writing the above in the standard form we get $7x^3 - 2x^2 - 5x + 6$ and $3x^2 + 6x - 9$

$$\begin{array}{r} 7x^3 - 2x^2 - 5x + 6 \\ 3x^2 + 6x - 9 \\ x^3 - 4x + 2 \\ \hline 8x^3 + x^2 - 3x - 1 \end{array}$$

$\therefore$ The sum is $8x^3 + x^2 - 3x - 1$

4.3.1. (b) Addition of polynomials of unlike terms :

We cannot add unlike terms. In this case we simply indicate the operation of addition.

Example 14 :

Add: $7a$, $5b$, $3x$ and $6y$

Solution :

Since all the terms given are unlike terms we just write all these terms putting the + sign in between them.

The sum is $7a + 5b + 3x + 6y$

Note: The answer cannot be simplified as in the case of addition of like terms.

Example 15 :

Add: $5m + 2n$ and $3x - 2y$

Solution :

Since the two polynomials have no like terms, we just add the terms of the polynomials as such.

$5m + 2n + 3x + (-2y) = 5m + 2n + 3x - 2y$

$\therefore$ The sum is $5m + 2n + 3x - 2y$

Exercise 4.3

1. ***Add the following terms:***
   (a) 2x, 8x and 5x (b) 5y, 3y, − 2y and − 4y (c) $4m^2$, $-3m^2$, $-6m^2$ and m^2
   (d) 3a, − 4b and 6p
2. ***Add the following polynomials:***
   (a) 3a + 7b and 2a − 3b
   (b) 8x + 6 and 4x − 3
   (c) m − 2n and 3p + 4q
   (d) 4a − 3b − 4c; 3b − 6c − 5a and 3a + 4b + 7c
   (e) $4x^2 - 3x + 7$; $8 + 4x - x^2$ and $5x - 2x^2 - 10$
   (f) $3m - m^2 + 1$; $-6m - 9$ and $m^2 - 4m + 7$
   (g) 2p − 3q + 4r ; 3p − 10q − 5r and 9p + 5q + r
   (h) 5a − 3b + 7c and 2x + y − z

4.3.2 Subtraction of polynomials :

To subtract a polynomial from another polynomial,

(a) rewrite the polynomials in two lines such that the lower line is the polynomial to be subtracted and the like terms of both the polynomials are one below the other.

(b) then change the sign of each term in the lower line from plus to minus [+ to −] and minus to plus [− to +]

(c) now do the sum as we have done in addition.

Example 16 :

Subtract (a) $2x + 3$ from $5x + 7$ (b) $5a - 2b$ from $3b - 4a$ (c) $p + 2q$ from $3m - 5n$

Solution :

(a) Subtract $2x + 3$ from $5x + 7$

The polynomial to be subtracted is $2x + 3$.

$\therefore$ We should write it in the second line.

$$\begin{array}{l} 5x + 7 \\ 2x + 3 \\ \underline{(-)\quad(-)} \quad \text{(Changing the sign)} \\ 3x + 4 \quad \text{(Adding the polynomials)} \end{array}$$

$\therefore$ The difference is $3x + 4$.

Note: After changing the sign, we should not consider the original signs.

(b) Subtract $5a - 2b$ from $3b - 4a$

$3b - 4a$ is to be written as $-4a + 3b$ (alphabetically)

$-4a + 3b$

$5a - 2b$

(–) (+)

$-9a + 5b$ ∴ The difference is $-9a + 5b$

(c) Subtract $p + 2q$ from $3m - 5n$

Since the two polynomials have no like terms just add the polynomials after changing the sign of each term of the polynomial to be subtracted.

$p + 2q$ will be changed as $-p - 2q$. Add the polynomials $3m - 5n$ and $-p-2q$

∴ The difference is $3m - 5n - p - 2q$.

Example 17 :

Subtract $10x - 3y - 5z$ from $6y - 7z + 12x$

Solution :

$12x + 6y - 7z$ (Written in order)

$10x - 3y - 5z$

(–) (+) (+)

$2x + 9y - 2z$ ∴ The difference is $2x + 9y - 2z$

Example 18 :

Subtract $2x^2 - 7 + 5x$ from $6x - 5 - x^2$

Solution :

(Rewrite the given polynomials in the standard form).

$-x^2 + 6x - 5$

$2x^2 + 5x - 7$

(–) (–) (+)

$-3x^2 + x + 2$ ∴ The difference is $-3x^2 + x + 2$

Example 19 :

Subtract $6x - 5y + z$ from the sum of $5x - 7y + 10z$ and $-3x + 5y + 6z$.

Solution :

[Do the addition part first and then the subtraction part]

(Add)

$5x - 7y + 10z$

$-3x + 5y + 6z$

$2x - 2y + 16z$

(Subtract)

$2x - 2y + 16z$

$6x - 5y + z$

(–) (+) (–)

$-4x + 3y + 15z$

∴ The answer is $-4x + 3y + 15z$

Exercise 4.4

1. *Subtract the first polynomial from the second polynomial.*

   (a) $7a + 6b$; $10a + 9b$ *(b) $2x - 3y$; $3x + y$* *(c) $3m - 4$; $5m + 7$*

   (d) $r - 2$; $3p + 5q$ *(e) $x^2 - 3x - 6$; $2x^2 + 5x - 4$*

   (f) $a - 2c - 4b$; $5c - 3b + 2a$ *(g) $x - 5y - 9z$; $10z + 2x - 8y$*

2. *Subtract $x + y + z$ from the sum of $x + y - 2z$ and $2x - y + z$*
3. *From the sum of $3a - b + c$ and $-b - c$, subtract $5a - b - c$*

4.3.3 Multiplication of monomials :

We know that multiplication is the simplified form of addition. We have already learnt that the multiplication of numbers and the principles of multiplying terms with + and – signs.

Recall:

1. **Positive Term × Positive Term = Positive Term**
2. **Negative Term × Negative Term = Positive Term**

   That is the product of terms with the same signs is positive.
3. **Positive Term × Negative Term = Negative Term**
4. **Negative Term × Positive Term = Negative Term.**

That is the product of terms with different signs is negative.

Now let us study the multiplication of monomials. Consider the product of a^2 and a^3

The product of a^2 and a^3 is $a^2 \times a^3$

We know that $a \times a = a^2$; $a \times a \times a = a^3$

$$\therefore a^2 \times a^3 = (a \times a) \times (a \times a \times a)$$
$$= a \times a \times a \times a \times a = a^5$$

or simply , $a^2 \times a^3 = a^{2+3} = a^5$

Consider $m^3 \times m^4 \times m$

$$m^3 \times m^4 \times m = (m \times m \times m) \times (m \times m \times m \times m) \times m$$
$$= m \times m \times m \times m \times m \times m \times m \times m$$
$$= m^8$$

or simply, $m^3 \times m^4 \times m = m^{3+4+1} = m^8$

From the above examples we understand that **to multiply variables, the powers of like variables are added.**

Example 20:

Multiply the following:

(a) $y \times y^4$ (b) $a^3 \times a^5 \times a^2$ (c) $x^2 \times x^5 \times x^7$

Solution:

(a) $y \times y^4 = y^1 \times y^4 = y^{1+4} = y^5$

(b) $a^3 \times a^5 \times a^2 = a^{3+5+2} = a^{10}$

(c) $x^2 \times x^5 \times x^7 = x^{2+5+7} = x^{14}$

Now let us consider monomials with integral coefficients.

To multiply monomials of this type, use the following steps.

Step 1 : Multiply the integers.
Step 2 : Multiply the like variables. (add the powers of like variables)

Example 21 :
Find the product of:
(a) $2a^2$ and $5a$ (b) $-3x^3$ and $-4x^2$ (c) $11p^2$ and $-6p^5$
(d) $-2m^3$ and $7m^5$ (e) $3y^3$, $-6y^2$ and $5y$
Solution:

(a) $2a^2 \times 5a = (2 \times 5) \times (a^2 \times a^1)$
$= 10 \times a^{2+1} = 10a^3$

(b) $-3x^3 \times (-4x^2) = (-3 \times -4) \times (x^3 \times x^2)$
$= 12x^{3+2} = 12x^5$

(c) $11p^2 \times (-6p^5) = [(11 \times -6) \times (p^2 \times p^5)]$
$= -66 \times p^{2+5} = -66p^7$

(d) $-2m^3 \times 7m^5 = (-2 \times 7) \times (m^3 \times m^5)$
$= -14\, m^{3+5}$
$= -14m^8$

(e) $3y^3 \times (-6y^2) \times 5y = [(3 \times (-6) \times 5)] \times (y^3 \times y^2 \times y^1)$
$= -90 \times y^{3+2+1} = -90y^6$

Exercise 4.5

Find the product of the following terms:

(1) x^2 and x^5 ***(2) a^3 , a and a^6*** ***(3) $6p^6$ and $10p^{10}$***
(4) $4y^2$, $3y$ and 7 ***(5) $-5c^4$ and $-9c^6$*** ***(6) $7x^2$ and $-11x^4$***
(7) $-8z$ and $4z^4$ ***(8) $-4m^3$, $-3m^4$ and -2*** ***(9) $-3n^2$, $4n^5$ and $-n$***
(10) $5x^8$, $6x^2$ and $3x^{10}$

4.3.4 Division of monomials :

We are already familiar with the division of integers. Recall the following facts that we have learnt in the chapter 'Number Work'.

1). $\frac{\text{Positive term}}{\text{Positive term}}$ = Positive term **2). $\frac{\text{Negative term}}{\text{Negative term}}$ = Positive term**

3). $\frac{\text{Positive term}}{\text{Negative term}}$ = Negative term **4). $\frac{\text{Negative term}}{\text{Positive term}}$ = Negative term**

Now let us study the division of monomials.

Consider $x^4 \div x^2$

$$x^4 \div x^2 = \frac{x^4}{x^2} = \frac{x \times x \times x \times x}{x \times x} = x^2$$

$$\text{or simply } \frac{x^4}{x^2} = x^{4-2} = x^2$$

Consider $m^5 \div m^4$

$$m^5 \div m^4 = \frac{m^5}{m^4}$$

$$= \frac{m \times m \times m \times m \times m}{m \times m \times m \times m} = m$$

$$\text{or simply } \frac{m^5}{m^4} = m^{5-4} = m$$

From the above examples we understand that **to divide variables, the powers of like variables are subtracted.**

To divide the monomials, use the following steps.

Step 1 : Divide the integers.

Step 2 : Divide the like variables. (subtract the power of the like variable in the denominator from that of the numerator)

Example 22 :

Divide : (a) a^8 by a^5 (b) $-6m^{10}$ by $-m^6$ (c) $-28x^5$ by $7x^3$

(d) $9p^{12}$ by $-9p^5$

Solution :

(a) $\frac{a^8}{a^5} = a^{8-5} = a^3$

(b) $\frac{-6m^{10}}{-m^6} = 6m^{10-6}$ (since $\frac{-6}{-1} = 6$)

$= 6m^4$

(c) $\frac{-28x^5}{7x^3} = \frac{-28}{7} \times \frac{x^5}{x^3}$

$= -4 \times x^{5-3} = -4x^2$

(d) $\frac{9P^{12}}{-9P^5} = \frac{9}{-9} \times \frac{p^{12}}{p^5}$

$= -1 \times p^{12-5} = -p^7$

Example 23 :

Simplify : (a) $12x^2 \times (-8x^5) \div 16x^3$

(b) $-7m^3 \times 10m^7 \div (-5m^4)$

Solution:

(a) $12x^2 \times (-8x^5) \div 16x^3 = \frac{12x^2 \times (-8x^5)}{16x^3}$

$$= \frac{12 \times (-8) \times x^{2+5}}{16 \times x^3}$$

$$= \frac{-96x^7}{16x^3} = -6x^{7-3} = -6x^4$$

(b) $-7m^3 \times 10m^7 \div (-5m^4) = \frac{-7m^3 \times 10m^7}{-5m^4}$

$$= \frac{-7 \times 10 \times m^{3+7}}{-5m^4}$$

$$= \frac{-70m^{10}}{-5m^4} = 14m^{10-4} = 14m^6$$

Exercise 4.6

1. *Divide:*
   (a) a^7 *by* a^2 *(b)* $-m^{18}$ *by* $-m^{12}$ *(c)* x^{21} *by* $-x^{15}$
   (d) $-y^{15}$ *by* y^6 *(e)* $6p^3$ *by* $3p^2$ *(f)* $-24c^{14}$ *by* $6c^3$
2. *Simplify the following:*
   (a) $\frac{16n^5}{4n^3}$ *(b)* $-9c^6 \div (-3c^5)$ *(c)* $3p^8 \times (-6p^7) \div 18p^{11}$
   (d) $\frac{-6c^3 \times (-10c^4)}{-12c}$ *(e)* $\frac{5x^{11} \times 6x^9}{15x^8}$

4.4 Finding the value of an expression :

We know that the literals represent numbers.

Consider the expression $5x - 3$

If $x =$ 1 the value of $5x - 3 = 5 \times 1 - 3 = 5 - 3 = 2$

If $x =$ 0 the value of $5x - 3 = 5 \times 0 - 3 = 0 - 3 = -3$

If $x = -2$ the value of $5x - 3 = 5 \times (-2) - 3 = -10 - 3 = -13$

From the above examples, if the value of x changes, the value of the expression also changes. Hence the value of an expression depends on the value of its variables.

The method of replacing the variables by numerical values is called substitution.

Example 24 :

Find the value of $4x - 5$ if $x = 3$.

Solution :

Substitute $x = 3$ in the expression $4x - 5$

$\therefore 4x - 5 = 4(3) - 5$

$= 12 - 5$

$= 7$

Example 25 :

If $a = 2$ and $b = -1$ find the value of $2a + 3b$.

Solution :

Substitute $a = 2$ and $b = -1$ in $2a + 3b$

$\therefore 2a + 3b = 2(2) + 3(-1)$

$= 4 - 3$

$= 1$

Example 26:

If $a = 5$, $b = 3$ find the value of $a^2 + ab$

Solution:

Substitute $a = 5$ and $b = 3$ in $a^2 + ab$

$\therefore\ a^2 + ab = a \times a + ab$

$= 5 \times 5 + 5 \times 3$

$= 25 + 15$

$= 40$

Example 27 :

The length of a rectangle is $7cm$ and its breadth is $5cm$. Find its area and perimeter if area, $A = lb$ sq. units and perimeter, $P = 2l + 2b$ units.

Solution :

Length , $l = 7$cm

Breadth , $b = 5$cm

a) Area of the rectangle , $A = l \times b$ sq. units

$= 7 \times 5$ (Substituting the values of l and b)

$= 35$

Hence Area, $A = 35$ sq.cm.

b) Perimeter of the rectangle , $P = 2l + 2b$ units

$= 2 \times 7 + 2 \times 5$

$= 14 + 10$

$= 24$

$\therefore$ Perimeter, $P = 24\ cm$

Exercise 4.7

1. *If $x = 1$, $y = 2$ find the values of the following expressions:*
   (a) $x + y$ (b) $3x + 4$ (c) $5y - 3$ (d) $4x - 3y + 5$
   (e) $xy + 3y - 9$ (f) $x^2 + y^2$
2. *If $a = 3$, $b = -2$ and $c = 1$ find the values of the following expressions :*
   (a) $a + b + c$ (b) $ab - c$ (c) $a^2 - b^2$ (d) $abc + 5$
   (e) $a^2 - 2b^2 + 3c^2$
3. *The measure of the side of a square room is 6m. Find its area and perimeter.*
   (Area , $A = a^2$ sq. units and perimeter, $P = 4a$ units)
4. *Area of a triangle is given by , $A = \frac{1}{2}bh$ sq. units. If $b = 12cm$ and $h = 7cm$ find the area of the triangle.*

4.5 Equations :

A Statement containing numbers or expressions in which both the sides are equal is called an equation. Equations are useful for relating variables and numbers.

$2 = 2$; $17 = 2 + 15$; $7 = x$; $t + 3 = 8$ and $3n + 16 = 100$ are some examples for equations.

That is , a statement involving the symbol "=" (is equal to) is called an equation.

Consider the following statements:

a) x added to 7 is 18 can be written as $x + 7 = 18$

b) 3 subtracted from m gives 6 can be written as $m - 3 = 6$

c) 8 times a is 40 can be written as $8a = 40$

d) y divided by 4 is 25 can be written as $\frac{y}{4} = 25$

e) Sum of two numbers a and b is 22 can be written as $a + b = 22$

f) A number n multiplied by itself is 49 can be written as $n \times n = 49$ or $n^2 = 49$

We observe that the above statements involve one or more unknowns. We represent these unknowns by variables. Also we observe that each statement is connected by an equality sign. Hence each is called an equation.

An equation has two sides. The side on the left of the equality sign is the Left Hand Side (Written as LHS) and the side on the right of the equality sign is the Right Hand Side (Written as RHS).

For example in the equation $x + 7 = 18$, $x + 7$ is the LHS and 18 is the RHS.

4.5.1	**Simple equations**
4.5.2	**Framing simple equations**
4.5.3	**Solving simple equations**

4.5.1 Simple equations:

Consider the following equations:

1) $x + 3 = 9$

This equation has only one variable x and the power of x is one.

2) $2m - 5 = 7$

In this equation the variable is m and its power is one.

3) $2a + 3b = 20$

In this equation there are two variables a and b. The power of a and b are one.

4) $y^2 = 64$.

In this equation there is only one variable y but the power of y is two.

Here $x + 3 = 9$ and $2m - 5 = 7$ are called simple equations because they have only one variable whose power is also one.

Hence an equation which has only one variable whose power is one is called a simple equation.

Example 28 :

Choose the simple equations from the following:

(a) $x + 5 = 11$ (b) $3a - 1 = 14$ (c) $x + y = 15$ (d) $2m - 3 = 15$

(e) $6n = 30$ (f) $y^2 + 1 = 50$

Solution :

(a) $x + 5 = 11$, (b) $3a - 1 = 14$, (d) $2m - 3 = 15$ and (e) $6n = 30$ are simple equations.

Note: The equation $x + y = 15$ has two variables x and y. Hence it is not a simple equation.

The equation $y^2 + 1 = 50$ has only one variable y. But the power of y is 2. Hence it is not a simple equation.

4.5.2 Framing simple equations :

Mathematical statements involve relations among known and unknown quantities. These relations are often stated in words and hence we call these problems (statements) as word problems. We rewrite these word problems as equations by using variables for unknown quantities. We should follow a few important steps in framing equations.

For example consider the statement, 'when 7 is added to a number the sum is 16 '.

Step 1: Read this problem carefully. Note down what is given and what is required.

In this statement (problem) the numbers given are 7 and 16.

The required number is the unknown number that we have to find out.

Step 2: Denote the unknown number by any one of the alphabets such as a , b , c , x, y , z etc.

In this problem let us denote the unknown number by x.

Step 3: Now rewrite the word problem using numbers and variables.

The given statement is ' when 7 is added to a number the sum is 16 '.

It can be written as $x + 7 = 16$.Hence the simple equation is $x + 7 = 16$

Note: While framing equations we need not find the value of the unknown unless it is asked in the problem to find the value.

Example 29 :

The sum of an unknown number and 10 is 21, frame an equation.

Solution :

Let the unknown number be x.

One number is 10 and the sum of two numbers is 21.

$\therefore$ The simple equation is $x + 10 = 21$

Example 30 :

Total number of students in a class is 70. If the number of boys is 37, frame an equation to find the number of girls.

Solution :

Let the number of girls be x.

The number of boys is 37.

The total number of students is 70.

$\therefore$ The simple equation is $x + 37 = 70$

Example 31 :

4 subtracted from 3 times of a number is 32. Frame an equation.

Solution :

Let the unknown number be x.

3 times of x is $3 \times x = 3x$

when 4 is subtracted from $3x$ we get 32

$\therefore$ The simple equation is $3x - 4 = 32$

Exercise 4.8

1. Which of the following are simple equations?

(a) $x + 7 = 10$ *(b)* $a + b = 17$ *(c)* $2p = 6$ *(d)* $m^2 = 16$ *(e)* $\frac{k}{3} = 5$

(f) $a^2 - 1 = 15$ *(g)* $2y - 7 = 25$

2. ***Frame simple equations for the following statements:***
   (a) The sum of a number and 12 is 27.
   (b) The sum of 5x and 1 is 16
   (c) Subtract y from 8 to get 6.
   (d) 2 times of a number added to 5 is 25.
   (e) 7 times p is equal to 42.
   (f) z when divided by 3 is equal to 2.
   (g) 20 subtracted from a number is 7.
   (h) The cost of 6 pens is Rs 72.
   (i) Sharmila's age is 9. Sum of her age and her mother's age is 42.
   (j) In a hall there are 55 plastic chairs and some wooden chairs. Total number of chairs is 95.

4.5.3. Solving simple equations :

4.5.3(a) Solution of an equation :

In an equation having one variable the number that makes the equation true is called the solution of the equation. When we substitute the number for the variable the equation is satisfied.

For example, consider the simple equation $x - 5 = 3$

In this equation the L.H.S is $x - 5$ and the R.H.S is 3.

Here x is the variable (unknown)

When we substitute $x = 8$ the LHS becomes $8 - 5 = 3$.

Hence for $x = 8$ the LHS is equal to the RHS.

If we substitute any other value for x in this equation other than 8 we observe that the LHS is not equal to the RHS

$\therefore$ $x = 8$ is the solution of the equation $x - 5 = 3$

Hence

The value of the variable for which the LHS of the equation is equal to the RHS is called the solution of the equation.

In other words the value of the variable which satisfies the equation is called the solution of the equation.

4.5.3(b) Solving a simple equation by Trial and Error Method :

Consider the simple equation $x + 7 = 10$.

Here the LHS is $x + 7$ and the RHS is 10.

Substitute values starting from zero till the LHS is equal to the RHS.

		LHS	RHS	
If	$x = 0$,	$\underline{0} + 7 = 7$	10	LHS $\neq$ RHS
If	$x = 1$,	$\underline{1} + 7 = 8$	10	LHS $\neq$ RHS
If	$x = 2$,	$\underline{2} + 7 = 9$	10	LHS $\neq$ RHS
If	$x = 3$,	$\underline{3} + 7 = 10$	**10**	**LHS = RHS**

We observe that when $x = 3$ both the LHS and the RHS are equal. For all other values of x the LHS is not equal to the RHS.

$\therefore$ $x = 3$ is the solution (or the root) of the equation $x + 7 = 10$

Example 32 :

Find the solution of the equation $5y = 20$ by the Trial and Error Method.

Solution:

The given equation is $5y = 20$

	LHS	**RHS**	
If y = 0,	$5 \times 0 = 0$	20	LHS $\neq$ RHS
If y = 1,	$5 \times 1 = 5$	20	LHS $\neq$ RHS
If y = 2,	$5 \times 2 = 10$	20	LHS $\neq$ RHS
If y = 3,	$5 \times 3 = 15$	20	LHS $\neq$ RHS
If y = 4,	**$5 \times 4 = 20$**	**20**	**LHS = RHS**

$\therefore$ y = 4 is the solution of the equation.

Note: In this method only after a certain number of trials we get the solution of the equation. Hence we call this method as Trial and Error method. We observe that this method takes time and is not always possible to find the solution easily.Therefore we use a better method for solving an equation.

4.5.3(c) Solving a simple equation by the method of Transposition :

In a balance if we put equal weights in both the pans we observe that the two pans remain in the same level. If we remove equal weights from both the pans, the pans still remain the same. Thus if we add equal weights or remove equal weights from both the pans, the pans remain in the same level.

In the same way, if we add the additive inverse of the known quantity on both sides of the equation then the value of the LHS and the RHS will be equal.

Example 33 :

Solve : $x - 7 = 3$

Solution :

In this equation the additive inverse of the known quantity -7 is 7.

Now adding 7 on both sides we get,

$x - 7 + 7 = 3 + 7$

$\therefore$ $x = 10$ is the solution.

Verification:

Substitute $x = 10$ in the equation $x - 7 = 3$

$\therefore$ LHS becomes $10 - 7 = 3$

RHS = 3

$\therefore$ for $x = 10$, LHS = RHS

Example 34 :

Solve: $y + 6 = 18$

Solution:

The additive inverse of 6 is -6

Adding – 6 on both sides we get

$y + 6 + (-6) = 18 + (-6)$

i.e $y + 6 - 6 = 18 - 6$

$\therefore y = 12$ is the solution.

Note: Substitute $y = 12$ in the equation $y + 6 = 18$ and verify the result.

If we divide both sides of the equation by the coefficient of the variable then the value of the LHS and the RHS will be equal.

Example 35 :

Solve: $9x = 63$

Solution:

The coefficient of the variable x is 9. Dividing both sides by 9 we get

$$\frac{9x}{9} = \frac{63}{9}$$

$$x = 7$$

$\therefore x = 7$ is the solution.

Note : Verify the result by substituting $x = 7$ in the above equation.

If we multiply both sides of the equation by the multiplicative inverse of the coefficient of the variable then the values on both sides of the equation will be equal.

Example 36 :

Solve : $\frac{m}{5} = 11$

Solution:

$\frac{m}{5} = 11$ can be written as $\frac{1}{5} \times m = 11$

The coefficient of m is $\frac{1}{5}$

The multiplicative inverse of $\frac{1}{5}$ is 5.

Multiplying both sides of the equation by 5

We get

$$5 \times \frac{1}{5} \times m = 5 \times 11$$

$$m = 55$$

$\therefore m = 55$ is the solution of the equation.

Example 37 :

Solve $14x + 5 = -37$

Solution :

Step 1 : $14x + 5 + (-5) = -37 + (-5)$ [additive inverse of 5 is –5]

$14x + 5 - 5 = -37 - 5$

$\therefore 14x = -42$

Step 2 : $\frac{1}{14} \times 14x = \frac{1}{14} \times (-42)$ [multiplicative inverse of 14 is $\frac{1}{14}$]

$x = -3$

$\therefore x = -3$ is the solution of the equation

From the above examples, the following simple rules can be framed.

Rules for the method of Transposition:

Rule 1 : A positive constant in the LHS becomes a negative constant when it is taken to the RHS and vice versa.

Rule 2 : A negative constant in the LHS becomes a positive constant when it is taken to the RHS and vice versa.

Rule 3 : The coefficient of the variable in the LHS becomes the multiplicative inverse when it is taken to the RHS and vice versa.

Therefore in a simple equation keeping only the variable on one side of the equation and bringing the constant to the other side is called the method of Transposition.

Example 38 :

Solve $x + 4 = 7$

Solution :

$x + 4 = 7$

$x = 7 - 4$ ($+4$ in the LHS becomes -4 when it comes to the RHS by rule 1)

$\therefore x = 3$

Example 39 :

Solve $y - 15 = 23$

Solution :

$y - 15 = 23$

$y = 23 + 15$ (-15 in the LHS becomes $+15$ when it comes to the RHS by rule 2)

$\therefore y = 38$

Example 40:

Solve : $8y = 32$

Solution:

$8y = 32$

$y = 32 \times \frac{1}{8}$ (The coefficient 8 of y in the LHS becomes $\frac{1}{8}$ when it comes to the RHS by rule 3)

$\therefore y = 4$

Example 41:

Solve : $\frac{a}{11} = 5$

Solution:

$\frac{a}{11} = 5$

$a = 5 \times 11$ (The coefficient $\frac{1}{11}$ of a becomes 11 when it comes to RHS by rule 3)

$\therefore a = 55$

Example 42 :

Solve : $2x - 3 = 15$

Solution:

$$2x - 3 = 15$$
$$2x = 15 + 3 \text{ (by rule 2)}$$
$$2x = 18$$
$$x = 18 \times \frac{1}{2} \text{ (by rule 3)}$$
$$\therefore x = 9$$

Example 43:

Solve : $5x - 6 = 3x + 2$

Solution:

Note: To solve the above problem, the variable on the RHS is brought to the LHS and the constant term on the LHS is brought to the RHS by using the above rules.

$$5x - 6 = 3x + 2$$
$$5x = 3x + 2 + 6$$
$$5x - 3x = 8$$
$$2x = 8$$
$$x = 8 \times \frac{1}{2}$$
$$\therefore x = 4$$

Example 44 :

The sum of two numbers is 20. If one number is 15, find the other number.

Solution :

Let the unknown number be x.

The sum of two numbers = 20

One number is 15

$\therefore$ The simple equation is $x + 15 = 20$

$x = 20 - 15$ (By rule 1)

$x = 5$

$\therefore$ The other number is 5.

Example 45 :

The cost of 28 books is Rs. 560. Find the cost of each book.

Solution:

Let the cost of one book be Rs. x

$\therefore$ The cost of 28 books is $28 \times x = 28x$

But the cost of 28 books = Rs. 560 (given)

$\therefore$ The simple equation is $28x = 560$

To get the cost of one book, solve the simple equation

$28x = 560$

$x = 560 \times \frac{1}{28}$ (by rule 3)

$x = 20$

$\therefore$ The cost of each book is Rs. 20

Example 46 :

When 7 is subtracted from 5 times of a number we get 33. Find the number.

Solution :

Let the unknown number be x.

$\therefore$ 5 times of x is $5 \times x = 5x$.

When 7 is subtracted from $5x$ we get 33.

$\therefore$ The simple equation is $5x - 7 = 33$

Solving this simple equation,

$$5x - 7 = 33$$

$$5x = 33 + 7 \text{ (By rule 2)}$$

$$5x = 40$$

$$x = 40 \times \frac{1}{5} \text{ (By rule 3)}$$

$$x = 8$$

$\therefore$ The unknown number is 8.

Exercise 4.9

1. ***Find the solution of the following equations by the Trial and Error Method :***

   (a) $x + 3 = 5$ *(b)* $y - 15 = -12$ *(c)* $5m = 10$ *(d)* $\frac{a}{2} = 2$

   (e) $p + 3 = 4$

2. ***Solve the following equations:***

   (a) $x - 6 = 10$ *(b)* $m + 13 = -15$ *(c)* $a + 14 = 19$ *(d)* $\frac{y}{3} = 8$

   (e) $7x = 21$ *(f)* $2a - 6 = 30$ *(g)* $5x + 9 = 54$ *(h)* $7 + 4y = -5$

   (i) $3x + 1 = 2x + 5$ *(j)* $7y - 3 = 4y - 9$

   Frame simple equations for the following and find their solutions:

3. ***The sum of two numbers is 35. If one number is 20. Find the other number.***

4. ***When we subtract 18 from a number we get 12. Find the unknown number.***

5. ***The cost of 5 pens is Rs 75. Find the cost of one pen.***

6. ***When a number is divided by 11, the quotient is 8. Find the number.***

7. ***When 5 is subtracted from 7 times of a number we get 30. Find the number.***

THINGS TO REMEMBER

1. **A combination of numerals, variables and arithmetical operations such as + , − , × and ÷ is known as an ALGEBRAIC EXPRESSION.**
2. **In an algebraic expression if the power of each variable is a whole number then it is called a POLYNOMIAL EXPRESSION or simply a POLYNOMIAL.**
3. **(a)** A polynomial **expression containing only one term is called a MONOMIAL.**
   (b) A polynomial **expression containing two terms is called a BINOMIAL.**
   (c) A polynomial **expression containing three terms is called a TRINOMIAL.**
4. **The highest power of the variable in a polynomial of one variable is called the DEGREE of the polynomial.**
5. **A polynomial written in descending order of the powers of the variable is said to be in the STANDARAD FORM.**
6. **To multiply variables, the powers of like variables are added.**
7. **To divide variables, the powers of like variables are subtracted.**
8. **An equation which has only one variable whose power is one is called a SIMPLE EQUATION.**
9. **The value of the variable which satisfies the equation is called the SOLUTION of the equation.**
10. **In a simple equation keeping only the variable in the LHS of the equation and bringing the constant to the RHS is called the method of TRANSPOSITION**
    Rules for the Method of Transposition:
    (1) A positive constant in the LHS becomes a negative constant when it is taken to the RHS and vice versa.
    (2) A negative constant in the LHS becomes a positive constant when it is taken to the RHS and vice versa.
    (3) The coefficient of the variable in the LHS becomes the multiplicative inverse when it is taken to the RHS and vice versa.

UNIT TEST – 4

Part –I

Choose the correct answer : *(14 × 1= 14)*

1. *In $5x^2$, the coefficient of x^2 is*

   (a) 2 *(b) x* *(c) 5* *(d) x^2*

2. *$-3x^2y$ is a*

   (a) Trinomial *(b) monomial* *(c) binomial* *(d) multinomial*

3. *The sum of 2x and 3x is*

   (a) $6x^2$ *(b) 6x* *(c) $5x^2$* *(d) 5x*

4. *The like term of $-5x^2$ is*

   (a) $2x^2$ *(b) $-5y^2$* *(c) –5x* *(d) 2x*

5. *The degree of the polynomial $x^3 - 4x - 5$ is*

   (a) –4 *(b) 3* *(c) 1* *(d) –5*

Complete the following :

6. *$3x - 7x =$ _____*
7. *$a^3 \times a^5 =$ _____*
8. *The highest power of the variable in a polynomial is called the _____ of the polynomial.*
9. *The power of x in 3x is _____*
10. *$x^5 \div x^2 =$ _____*
11. *To multiply variables, the powers of like variables are _____*
12. *$3x - 2 = 5$ is a _____ equation.*
13. *The number of terms in the polynomial $6x^2 - 8xy + 9y^2$ is _____*
14. *A polynomial containing three terms is called a _____.*

Part – II

Answer in one or two steps: *(8 × 2 = 16)*

15. *Add : $2x^2$; $5x^2$; $3x^2$*
16. *Write the polynomial $2x^2 - 3x + x^3 - 7$ in the standard form.*

17. Using the terms given below form any two binomials :

$-2y$, $4y^2$, y^3 , -7

18. Subtract : $2x - 3y$ from $3x + y$

19. Simplify: $\frac{-16x^8}{4x^3}$

20. Find the value of $3y - 4$ if $y = 5$

21. Sum of an unknown number and 17 is 30. Frame an equation.

22. Solve : $y - 9 = 12$

Part – III

Answer in detail : *(4 × 5 = 20)*

23. Subtract $x^2 - 2x - 6$ from the sum of $2x^2 - 5x + 3$ and $x^2 + x - 5$

24. Simplify :

$5y^4 \times 8y^9 \div (-10y^6)$

25. When 4 is added to 5 times of a number we get 34. Find the number.

26. Add the following polynomials:

$4 - 3a + 7a^2$; $4a - 1 + 3a^2$ and $5a^2 - 3a - 7$

5. GEOMETRY

Geometry is a study of size, shape and properties of objects or figures.

The Sanskrit word **'Jyamithi'** means measure of the earth. English word **'Geometry'** is derived from the Greek word **'Geo'** meaning **'earth'** and **'metron'** meaning **'measuring'**. Euclid who lived in 300 B.C. was a distinguished Greek Mathematician known as the' Father of Geometry'.

5.1	**Symmetry**
5.2	**Tessellations**
5.3	**Transformations**
5.4	**Lines**
5.5	**Triangles**

5.1 Symmetry :

In our daily life we happen to see many things in nature which attract us. The attraction is due to the evenness or the uniformity of the things. Such qualities add beauty to the eyes of the observers.

For example, a person wearing a shirt with one hand in full and another hand in half makes us feel funny. This is because there is no uniformity or evenness in his shirt.

The quality of evenness or uniformity in the shape of the things is called symmetry.

For example, cut an apple into two equal halves. We observe that the two parts are in symmetry. The two most common types of symmetry are

(a) Mirror symmetry

(b) Rotational symmetry

5.1.1	**Line of symmetry**
5.1.2	**Mirror symmetry**
5.1.3	**Rotational symmetry**

5.1.1 Line of symmetry :

Take a rectangular piece of paper as in fig. 5.1(a). Fold the rectangular piece of paper into two halves as shown in fig. 5.1(b).

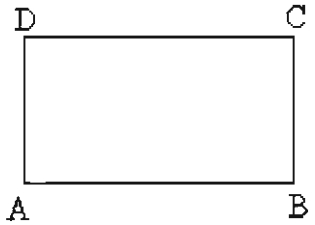


Fig. 5.1(a)

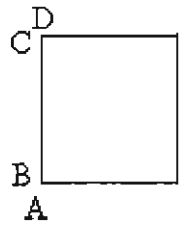


Fig. 5. 1 (b)

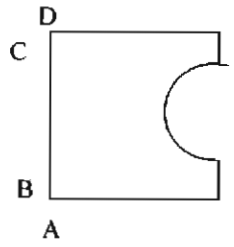


Fig. 5.1(c)

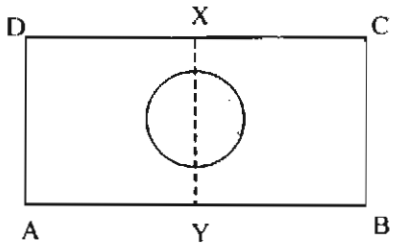


Fig. 5.1(d)

Now cut a piece of any design from the folded side of the paper as shown in fig. 5.1 (c) and remove the piece. On unfolding the paper, a design as shown in fig. 5.1 (d) is obtained. We find that the design obtained is the same on both the sides of the dotted line XY.

Thus whenever a line divides a given figure into two equal halves we say that the figure is symmetrical about the line. This line is called the line of symmetry or the axis of symmetry.

5.1.1 (a) A figure can have more than one line of symmetry :

In fig. 5.2, triangle ABC is an equilateral triangle and AD, BE and CF are the bisectors of $\angle A$, $\angle B$ and $\angle C$ respectively. Fold the figure along AD. The triangle ABC is divided into two equal halves ADB and ADC. Hence AD is a line of symmetry. Similarly BE and CF are also the other two lines of symmetry. Hence an equilateral triangle has three lines of symmetry.

$\therefore$ A figure can have more than one line of symmetry.

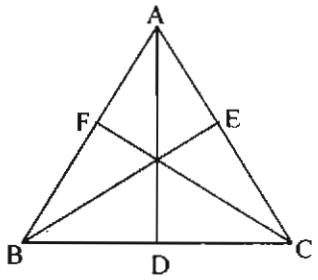


Fig. 5.2

5.1.1 (b) Every figure need not have a line of symmetry :

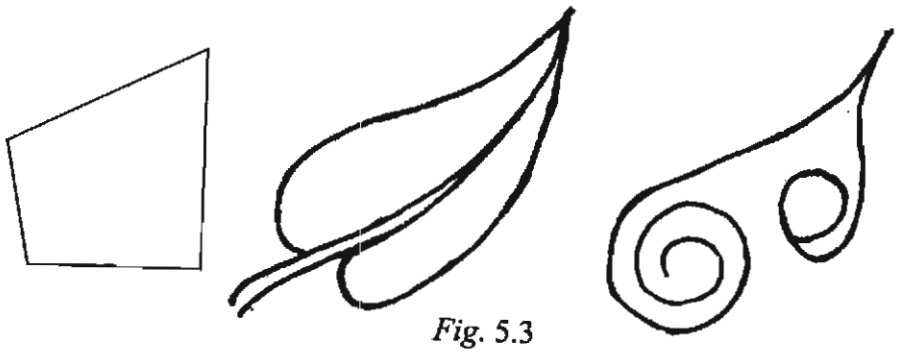


Fig. 5.3

Observe the above figures. We cannot divide the figures into two equal halves. Therefore they do not have a line of symmetry.

Hence it is not necessary for every figure to have a line of symmetry.

Example 1 :

The line of symmetry of any line segment is the perpendicular bisector of the line segment.

Solution :

In fig. 5.4, AB is a line segment. CD is the perpendicular bisector of AB meeting the line segment at O. Then OA = OB. CD divides the line segment into two equal halves.

$\therefore$ CD is the line of symmetry of the line segment AB.

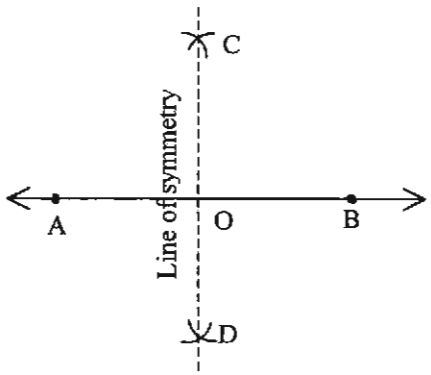


Fig. 5.4

Example 2 :

The line of symmetry of a given angle is the angle bisector of the angle.

Solution:

In fig. 5.5, OC is the angle bisector of $\angle AOB$ OC divides $\angle AOB$ into two equal halves.

$\therefore$ OC is the line of symmetry of $\angle AOB$.

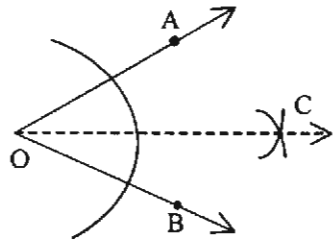


Fig. 5.5

Discuss : How many lines of symmetry does a circle have?

5.1.2 Mirror symmetry :

Take a thick smooth paper and draw a line AB on it. Mark a point P on one side of it. Erect a plane mirror along the line AB and look through it. Behind the mirror we can see another point P^1. P^1 is called the image of P. We observe that P and P^1 are symmetrical about the line AB. AB is called the line of symmetry.

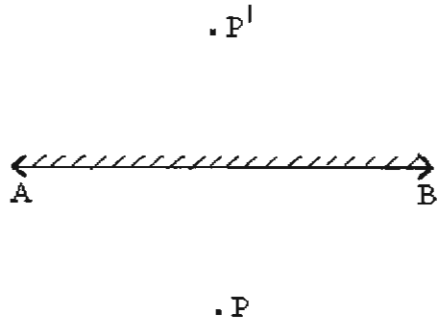
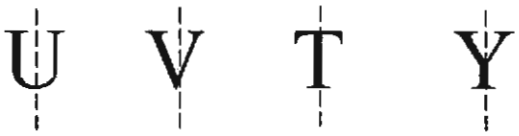


Fig. 5.6

Shapes or objects having one or more lines of symmetry are said to have mirror symmetry.

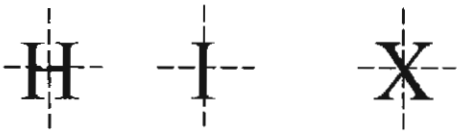
Some examples of mirror symmetry are given below :

The letters U , V, T and Y have mirror symmetry about a vertical line as shown below:



The letters D , E and C have mirror symmetry about a horizontal line as shown below :

The letters H , I and X have mirror symmetry about both horizontal and vertical lines as shown below :



Example 3 :

Mirror symmetry in a square.

Solution :

In fig. 5.7, ABCD is a square in which AC and BD are its diagonals. PQ is the perpendicular bisector of AB (as well as DC) and RS is the perpendicular bisector of BC (as well as AD). Each of AC , BD , PQ and RS divides the square ABCD into two equal halves. All these lines are the lines of symmetry of the square ABCD.

∴ The square has mirror symmetry.

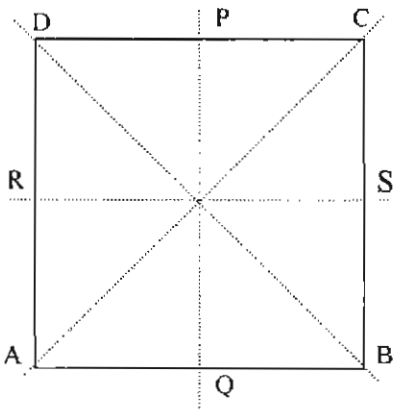


Fig. 5.7

Example 4 :

Mirror symmetry in a rectangle.

Solution:

In fig. 5.8, ABCD is a rectangle. Here AD = BC, PQ is the perpendicular bisector of AB and DC. PQ divides the rectangle ABCD into two equal halves. Therefore PQ is a line of symmetry.

Similarly RS is the perpendicular bisector of AD and BC. Therefore RS is another line of symmetry. (i.e.) There are two lines of symmetry for a rectangle.

∴ A rectangle has mirror symmetry.

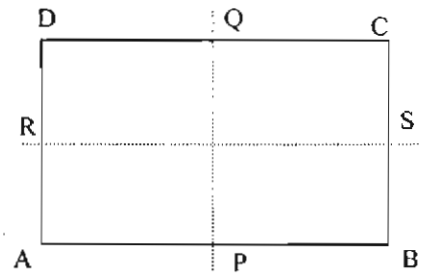


Fig. 5.8

Example 5 :

A parallelogram has no mirror symmetry.

Solution :

A parallelogram has no line of symmetry. Hence a parallelogram does not have mirror symmetry.

Example 6 :

Mirror symmetry in an isosceles triangle.

Solution :

The line of symmetry of an isosceles triangle is the angle bisector of the angle subtended by the two equal sides. ABC is an isosceles triangle in which AB = AC , AD is the bisector of $\angle A$. AD divides Δ ABC into two equal halves. AD is the line of symmetry of Δ ABC.

∴ An isosceles triangle has mirror symmetry.

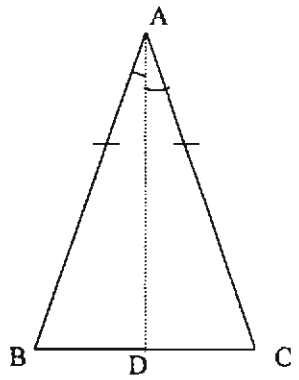


Fig. 5.9

Example 7 :

Mirror symmetry in an isosceles trapezium.

Solution:

The line of symmetry of an isosceles trapezium is the perpendicular bisector of the parallel sides. In fig. 5.10, ABCD is an isosceles trapezium in Which AD = BC and AB is parallel to DC. PQ is the perpendicular bisector of AB (as well as DC). PQ divides trapezium ABCD into two equal halves.PQ is the line of symmetry of the isosceles trapezium ABCD.

∴ An isosceles trapezium has mirror symmetry.

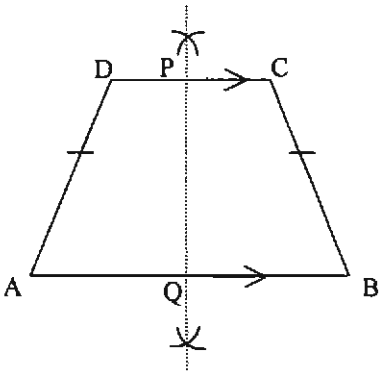


Fig. 5.10

Example 8 :

Mirror symmetry in a kite shaped figure :

Solution :

The line of symmetry for a kite shaped figure is the vertical diagonal. In fig. 5.11, ABCD is a kite in which AB = AD and BC = DC.The vertical diagonal AC divides the kite into two equal halves. AC is the line of symmetry of the kite ABCD.

∴ A kite shaped figure has mirror symmetry.

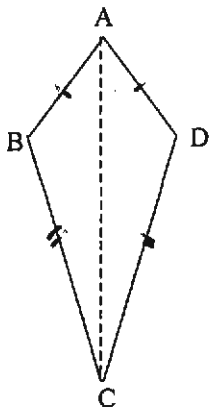


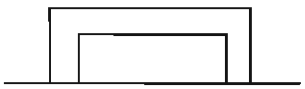
Fig. 5.11

Discuss: Does a circle have mirror symmetry?

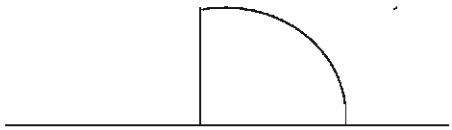
Exercise 5.1

1. In the following figures draw another part of the figure with respect to the given line of symmetry.

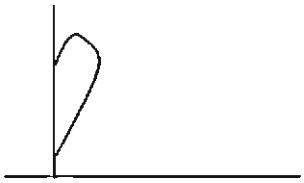
(a)



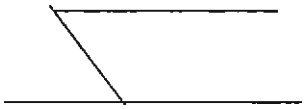
(b)



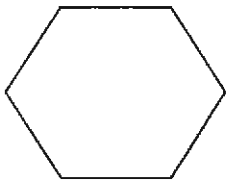
(c)



(d)



2. Draw all the lines of symmetry for the following figure.

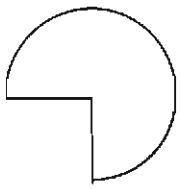


3. Draw the maximum number of lines of symmetry for the following figures and state the number of lines of symmetry.

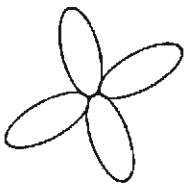
(a)

(b)

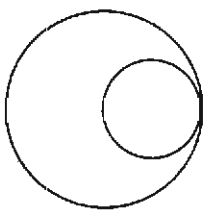
(c)



(d)



(e)



5.1.3 Rotational symmetry :

Take two cardboard sheets and cut off one equilateral triangle in each sheet such that both the triangles are identical. Place one triangle exactly over the other and put a pin through the centers of the figures. Rotate the top figure until it matches with the lower figure. We Notice that the triangle has been rotated through an angle 120°.

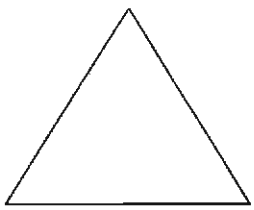


Fig. 5.12

Again rotate the top figure until it matches with the lower figure for the second time. Now we notice that the top figure has been rotated through an angle 240° from the original position.

Rotate the top figure for the third time to match with the lower figure. Now the top triangle has reached its original position after a complete rotation of 360°.

This kind of regularity is called the Rotational Symmetry. The fixed point about which the figure has to be rotated is called the centre of rotation.

How many turns have to be made before the top figure comes to its original position?

The top figure has been rotated three times to come back to its original position to match with the bottom figure.

Therefore it is said to have a rotational symmetry of order 3.

Since this triangle matches itself three times as it takes a complete rotation, it is said to have a rotational symmetry of order 3.

Note : A figure matches itself means after a turn the figure remains the same without any change in

the shape, design and pattern.

5. 1. 3 (a) Rotation of a hexagon through 60° :

For every 60° rotation, the hexagon matches with itself. A hexagon looks the same six times during a 360° rotation. Therefore the order of rotational symmetry for a hexagon is six.

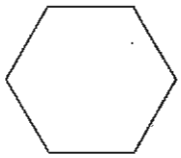


Fig. 5.13

5. 1. 3 (b) Rotation of a square through 90° :

For every 90° rotation, the square matches with itself. A square looks the same four times during a 360° rotation.

Therefore the order of rotational symmetry for a square is four.

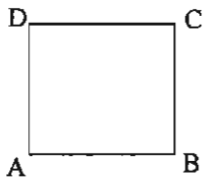


Fig. 5.14

5. 1. 3 (c) Rotation of an equilateral triangle through 120° :

For every 120° rotation, the equilateral triangle matches with itself. An equilateral triangle looks the same three times during a 360° rotation. Therefore the order of rotational symmetry for an equilateral triangle is three.

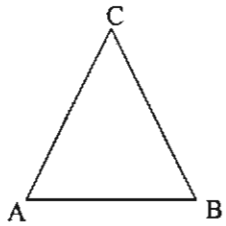


Fig. 5.15

5. 1. 3 (d) Rotation of a rectangle through 180° :

For every 180° rotation, the rectangle matches with itself. A rectangle looks the same two times during a 360° rotation. Therefore the order of rotational symmetry for a rectangle is two.

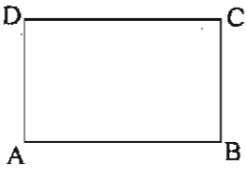


Fig. 5.16

Example 9 :

The figures having no lines of symmetry can have rotational symmetry.

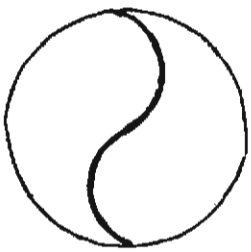


Fig. 5. 17

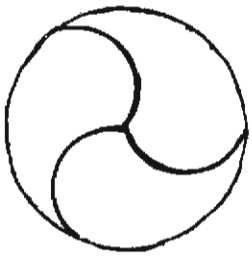


Fig. 5.18

The above figures have no lines of symmetry. But they both have a kind of regularity after a certain number of turns.

In fig. 5.17, the figure matches with itself two times as it takes a complete rotation. Therefore it is said to have rotational symmetry of order two.

In fig. 5.18, the figure matches with itself three times as it takes a complete rotation. Therefore it is said to have rotational symmetry of order three.

Example 10 :

Find the order of rotational symmetry for the fig.5.19

Solution :

For every 60° rotation the figure matches with itself. The figure looks the same six times during a 360° rotation. Therefore the order of rotational symmetry for the figure is six.

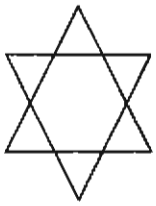


Fig. 5.19

Example 11:

Discuss the order of rotational symmetry for the fig.5.20

Solution :

For every 90° rotation the figure matches with itself.

The figure looks the same four times during a 360° rotation.

Therefore the order of rotational symmetry for the figure is four.

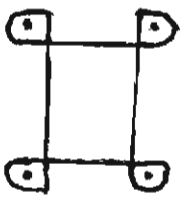


Fig. 5.20

Note 1 : Rotational symmetry of order one is the same as no rotational symmetry. The letters G , Q , J and L matches only once after a full 360° rotation. We say that they have a rotational symmetry of order one. So these letters have no rotational symmetry.

Note 2: For any object to have mirror symmetry there should be one or more lines of symmetry. But the objects having no line of symmetry can have rotational symmetry.

THINGS TO REMEMBER

1. **The quality of evenness or uniformity in the shape of the things is called symmetry.**
2. **A line of symmetry divides the figure into two equal halves.**
3. **A figure may have more than one line of symmetry.**
4. **Every figure need not have a line of symmetry.**
5. **The two most common types of symmetry are**
   (a) Mirror symmetry (b) Rotational symmetry.
6. **Shapes or objects having one or more lines of symmetry are said to have a mirror symmetry.**
7. **The figures having no lines of symmetry can have rotational symmetry.**
8. **When an object is rotated through 360°, the number of times it matches with itself is called the order of rotational symmetry of that object.**
9. **Objects having rotational symmetry of order one are said to have no rotational symmetry.**

Exercise 5.2

1. ***Find the number of lines of symmetry and the order of rotational symmetry for the following figures and enter the results in the table.***

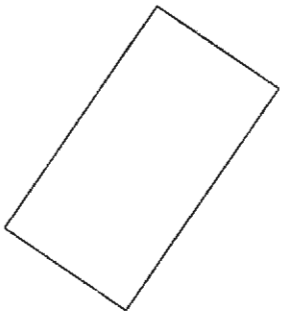


Fig. **(a)**

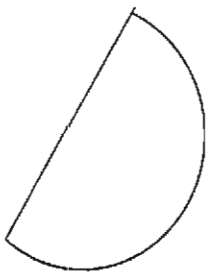


Fig. **(b)**

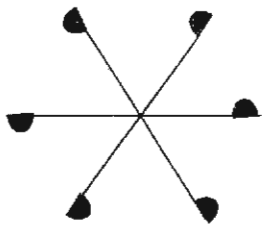


Fig. **(c)**

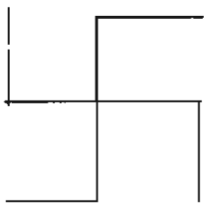


Fig. **(d)**

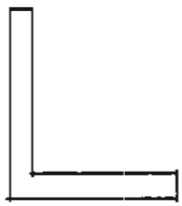


Fig. **(e)**

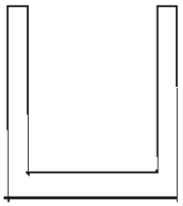


Fig. **(f)**

Fig.	*Lines of symmetry*	*Order of rotational symmetry*
(a)	*2*	*2*
(b)	—	—
(c)	—	—
(d)	—	—
(e)	—	—
(f)	—	—

2. Complete the following table :

Figure	*Lines of symmetry*	*Order of rotational symmetry*
(a) Equilateral triangle	*3*	*3*
(b) Isosceles triangle	—	—
(c) Square	—	—
(d) Rectangle	—	—
(e) Parallelogram	—	—

3. Find the number of lines of symmetry and the order of rotational symmetry for the following figures.

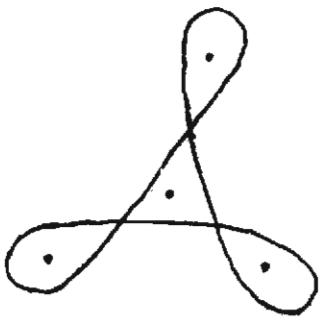


***Fig.* (a)**

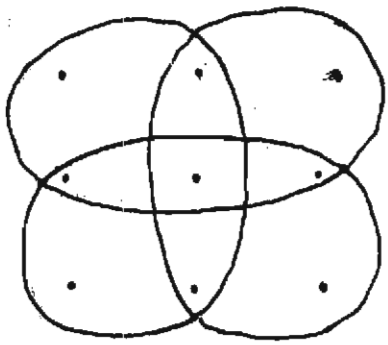


***Fig.* (b)**

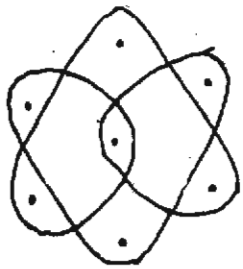


***Fig.* (c)**

5.2 Tessellations :

The dictionary meaning of the word 'tessellate' is to form or arrange small squares in a checkered or mosaic pattern. The word 'tessellate' is derived from the Greek word 'tessers' which means four.

A regular polygon has 3 or 4 or 5 or more sides in which all the sides and all the angles are equal. A regular polygon having 5 sides is called a Pentagon and a regular polygon having six sides is called a Hexagon.

Patterns which fill a plane are called tessellations.

A regular tessellation means a tessellation made up of regular polygons of same size.

Few examples are shown below :

Fig. 5.21 is a tessellation of equilateral triangles

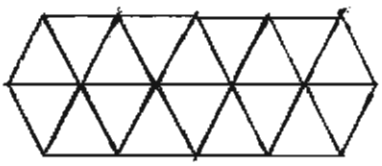


Fig. 5.21

Fig. 5.22 is a tessellation of squares.

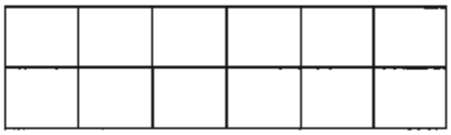


Fig. 5.22

Fig. 5.23 is a tessellation of hexagons.

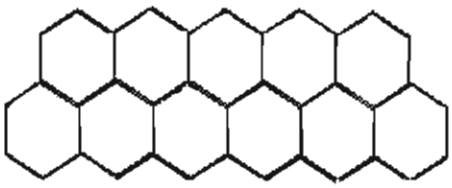


Fig. 5.23

Look at the above examples. We notice that the squares are lined up with each other as in the fig. 5.22. Here there is no gap between the tiles while filling the plane with the square shaped tiles.

From fig. 5.24 and fig. 5.25 we observe that the triangles and hexagons cannot be lined up with each other without leaving gaps between the tiles as in the case of squares.

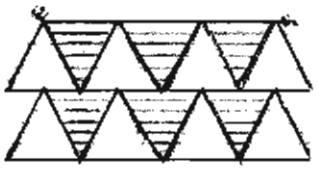


Fig. 5.24

Hence while arranging the tiles in the shape of triangles and hexagons to fill the plane a slide has to be used as in fig. 5.21 and fig. 5.23 to avoid leaving the gaps between the tiles.

From the above we observe that the tessellations are possible only if the tiles have the following shapes and interior angles.

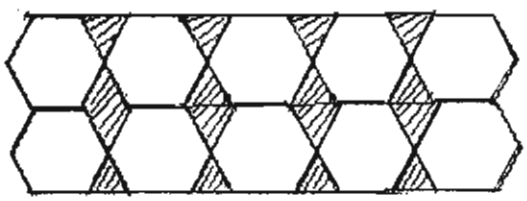


Fig. 5.25

Shapes	**Angle measure in degrees**
Equilateral triangle	60
Square	90
Hexagon	120

The squares, rectangles, triangles and hexagons are suitable shapes for tessellations.

Exercise 5.3

1. *What is meant by tessellation?*
2. *What are the different shapes that can be used for tessellation?*
3. *What are the interior angles of the figures that can be used for tessellation?*

5.3 Transformations :

Transformation is a change of position of an object on a plane.

Isometric Transformation :

A transformation is an isometric transformation if it preserves its distance. An isometric transformation can be either direct isometric transformation or opposite isometric transformation.

Direct Isometric Transformation :

An isometric transformation is direct if it preserves distance and orientation (direction).

Opposite Isometric Transformation :

An isometric transformation is opposite if it preserves distance but fails to preserve orientation (direction).

We now learn the four types of transformations. They are

(a) Translation
(b) Rotation
(c) Reflection
(d) Glide reflection

5.3.1 Translation
5.3.2 Rotation
5.3.3 Reflection
5.3.4 Glide reflection

5.3.1 Translation :

To translate an object means to move it from one place to another. Every translation has a direction and a distance. Here the object is the letter R.

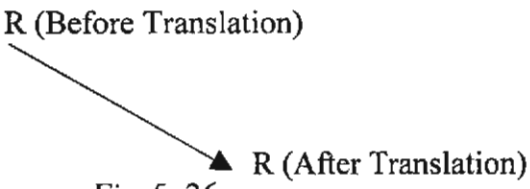


Fig. 5. 26

Example 12 :

Translation of triangle ABC.

Solution :

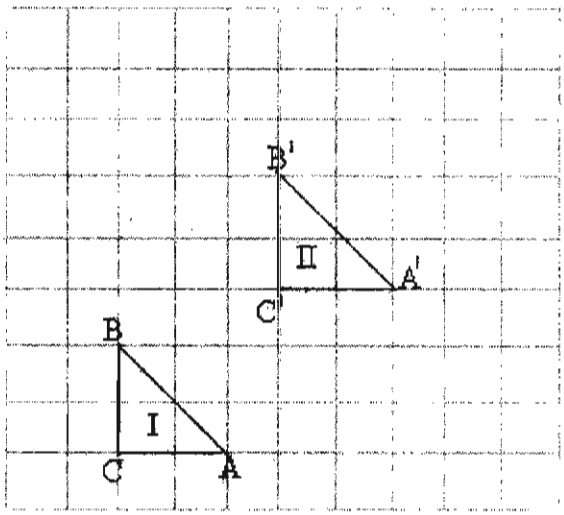


Fig. 5.27

In fig. 5.27, the triangle ABC has been moved from position I to position II as triangle $A^{'} B^{'} C^{'}$. We can see that A has moved 3 units (squares) along and 4 units (squares) up in order to coincide with $A^{'}$.

Can you say the change of position for B?

Yes. B has moved 3 units (squares) along and 4 units (squares) up in order to coincide with $B^{'}$.

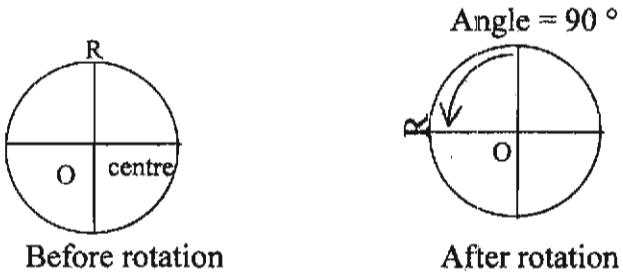
What is the change of position of C?

C has moved 3 units (squares) along and 4 units (squares) up in order to coincide with $C^{'}$. The triangle ABC is translated from position I to position II as triangle $A^{'} B^{'} C^{'}$.

Therefore under translation there is no change in shape and size of the object. But the position is changed.

5.3.2 Rotation :

To rotate an object means to turn it around. Every rotation has a centre and an angle of rotation.



Before rotation After rotation

Fig 5.28

Here the object of rotation is the letter R. In the above rotation, O is the centre of rotation and the angle of rotation is 90°.

From the above, we can say that under rotation there is no change in shape and size of the object. But the position is changed.

Activity : Rotate the above object R, through angles 180° , 270° and 360° about O and find out that the position of R in each rotation.

5.3.3 Reflection :

To reflect an object means to produce its mirror image.

Every reflection has a mirror line. A reflection of the object R is as shown in fig 5.29.

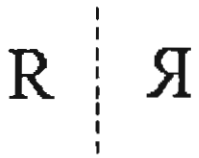


Fig.5.29

Before reflection | After reflection

Example 13 :

Find the reflection of the following :

(a) **B** (b) **K** (c) **S** (d) **E**

Solution :

(a)

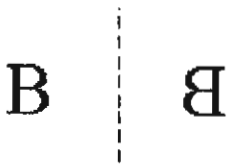


Fig. 5.30

Before reflection | After reflection

(b)

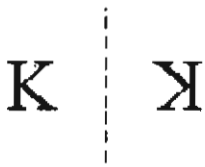


Fig. 5.31

Before reflection | After reflection

(c)

Fig. 5.32

Before reflection | After reflection

(d)

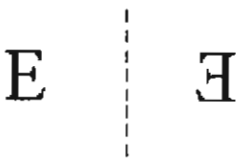
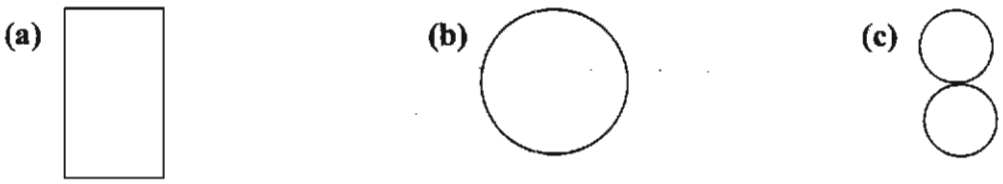


Fig. 5.33

Before reflection | After reflection

Example 14 :

Find the reflection of the following figures.



Solution :

(a)

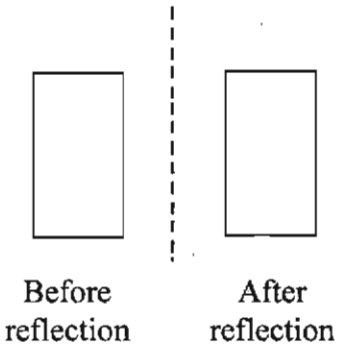


*Fig.*5.34

(b)

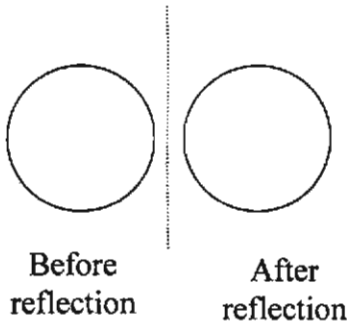


*Fig.*5.35

(c)

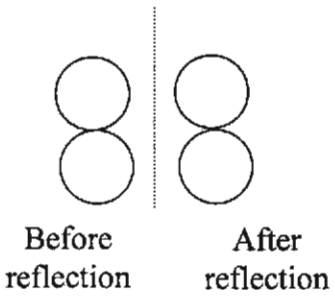


*Fig.*5.36

5.3.4 Glide reflection :

A glide reflection combines a reflection with a translation along the direction of the mirror line.

Glide reflection is the only transformation which involves more than one step.

In fig. 5.37, the object R is reflected first and then it is translated.

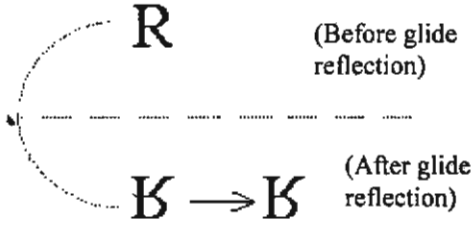


*Fig.*5.37

Note : From the above we observe that the size and shape of the objects are not changed under any of the four transformations. Therefore we say that **the size and the shape of the objects are invariant under transformations.**

THINGS TO REMEMBER

1. **The change of position of an object on a plane is called transformation.**
2. **An isometric transformation is direct if it preserves distance and orientation (direction).**
3. **An isometric transformation is opposite if it preserves distance but fails to preserve the orientation (direction).**
4. **There are four types of transformations. They are**

   (a) Translation (b) Rotation (c) Reflection (d) Glide reflection

5. **A glide reflection combines a reflection with a translation.**
6. **Under any of the four transformations there is no change in shape and size of the object but the position is changed.**

Exercise 5.4

1. ***What is meant by translation?***
2. ***What are the types of transformations?***
3. ***What is meant by rotation?***
4. ***What is meant by reflection?***
5. ***Reflect the following objects.***

(a) C *(b)* I *(c)* W *(d)* L

(e) *(f)*

6. ***What is glide reflection?***

5.4 Lines :

5.4.1	**Angles in intersecting lines**
5.4.2	**Adjacent angles on a line**
5.4.3	**Parallel lines**
5.4.4	**Transversal**

5.4.1 Angles in intersecting lines :

When two straight lines intersect at a point angles are formed. In fig. 5.38, the two intersecting lines are AB and CD. They intersect at O. There are four angles formed at the point O. The angles are $\angle AOC$, $\angle COB$, $\angle BOD$ and $\angle DOA$. The total angle formed at the point O is 360°. Among the four angles formed at the point O, two are acute and other two are obtuse.

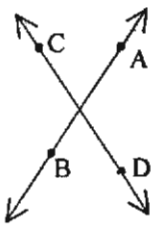


Fig. 5.38

But if the two intersecting lines are perpendicular to each other the four angles formed are right angles. (i.e. 90°) (Fig. 5.39).

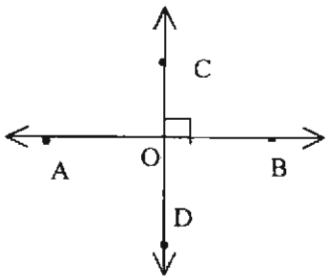


*Fig.*5.39

5.4.2 Adjacent angles on a line :

When a ray stands on a line two angles are formed. They are called adjacent angles on the line.

In fig. 5.40, the ray OC stands on the line AB. $\angle AOC$ and $\angle BOC$ are the two adjacent angles formed on the line AB. Here O is called the common vertex. OC is called the common arm. The arms OA and OB lie on the opposite sides of the common arm OC. Two angles are said to be adjacent on a line if they have a common vertex, a common arm and the other two arms are on the opposite sides of the common arm.

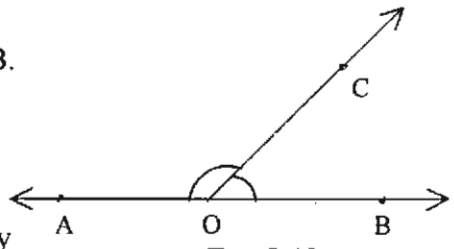


Fig. 5.40

5.4.2 (a) The sum of the adjacent angles on a line is 180° :

We have learnt that an angle whose measure is 180° is called the straight angle.

In fig. 5.41 , $\angle AOB = 180°$.

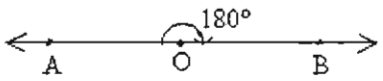


Fig. 5.41

In fig. 5.42 , OC stands on the line AB and $\angle AOC$ and $\angle BOC$ are adjacent angles. Since $\angle AOB$ is a straight angle whose measure is 180°, $\angle AOC + \angle BOC = 180°$.

$\therefore$ The sum of the adjacent angles on a line is 180°.

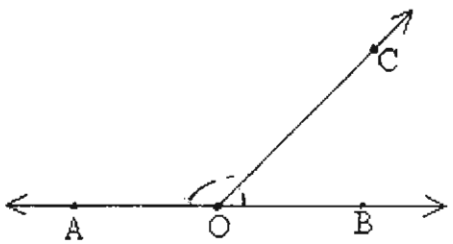


Fig. 5.42

5.4.2 (b) Angle at a point :

In fig. 5.43, there are four angles formed at the point O, which are $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$. The sum of the four angles formed is 360°.

(i.e.) $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$

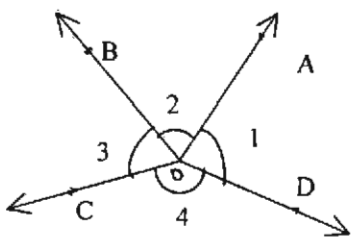


Fig. 5.43

5.4.2 (c) Vertically opposite angles are equal :

If two straight lines AB and CD intersect at a point O, then $\angle AOC$ and $\angle BOD$ form one pair of vertically opposite angles and $\angle AOD$ and $\angle BOC$ form another pair of vertically opposite angles.
When two lines intersect each other, the vertically opposite angles are equal.

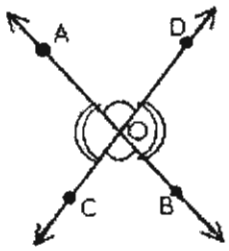


Fig. 5.44

From the fig 5.44,

$\angle AOC = \angle BOD$

and $\angle BOC = \angle AOD$

Example 15 :

Find the value of x in the figure.

Solution :

From fig. 5.45,

$50 + x = 180$ [since $\angle AOB$ is a straight angle]

$x = 180 - 50$

$x = 130°$

Therefore the value of x is 130°

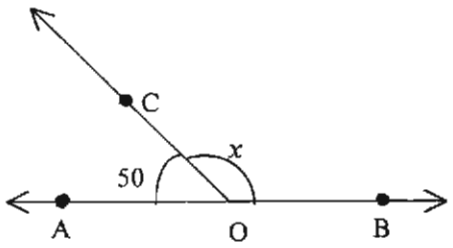


Fig. 5.45

Example 16 :

In fig. 5.46 , if $x = 110°$, find the value of y.

Solution :

$x + y = 180°$ [since $\angle AOB$ is a straight angle]

$110 + y = 180$

$y = 180 - 110$

$= 70$

Therefore the value of y is 70°.

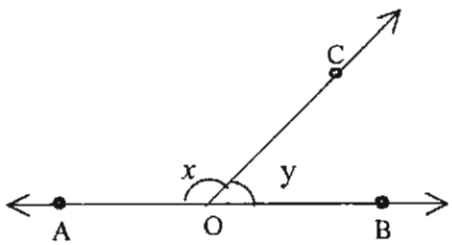


*Fig.*5.46

Example 17 :

In fig. 5.47 , find the value of x.

Solution :

$3x + 2x = 180$ (since $\angle PSQ$ is a straight angle)

$5x = 180$

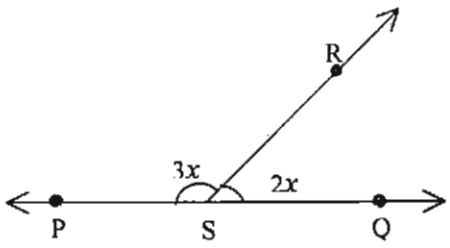


Fig. 5.47

$$x = \frac{180}{5}$$

$$= 36$$

Therefore the value of x is 36° .

Example 18 :

In fig. 5.48, determine the value of x and y.

Solution :

$x + 2x = 180$ (since $\angle BOA$ is a straight angle)

$$3x = 180$$

$$x = \frac{180}{3}$$

$$x = 60$$

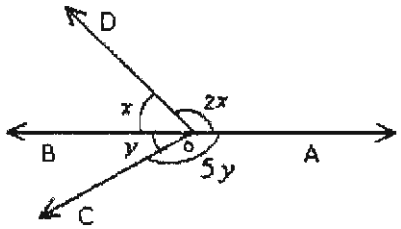


*Fig.*5.48

And $y + 5y = 180$ (since $\angle BOA$ is a straight angle)

$$6y = 180$$

$$y = \frac{180}{6}$$

$$y = 30$$

Therefore the value of x is 60° and the value of y is 30°.

Example 19 :

In fig. 5.49 , two lines AB and CD intersect at the point O.

Find the values of x and y.

Solution :

In fig. 5.49 ,

$\angle COA + \angle AOD = 180°$ (since $\angle COD$ is a straight angle)

$\angle COA = x$

$\angle AOD = 80°$

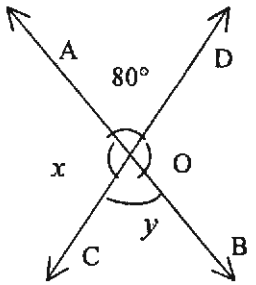


*Fig.*5.49

$\angle x + 80 = 180$

$\angle x = 180 - 80 = 100$

$\angle BOC + \angle COA = 180°$ (since $\angle BOA$ is a straight angle)

$\angle BOC = y$

$\angle COA = 100°$

$\therefore \quad \angle y + 100 = 180$

$\angle y = 180 - 100$

$\angle y = 80$

Therefore the value of x is 100° and y is 80°

Note : From the above example ,

we see that $\angle AOD = 80°$ [from fig.5.49]

$\angle BOC = 80°$

$\therefore \angle AOD = \angle BOC$

Hence the vertically opposite angles $\angle AOD$ and $\angle BOC$ are equal.

Similarly we can see that the other pair of vertically opposite angles $\angle COA$ and $\angle BOD$ are also equal.

Example 20 :

Find the value of x from the fig.5.50.

Solution :

We know that the sum of all the angles = 360°(since angle at a point is 360°)

(i.e) $2x + 2x + x + 2x + 45 = 360$

$7x + 45 = 360$

$7x = 360 - 45 = 315$

$x = \frac{315}{7}$

$x = 45$

Therefore the value of x is 45°.

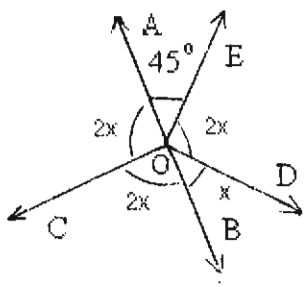


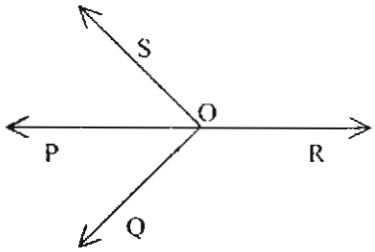
*Fig.*5.50

Exercise 5.5

1. From the figure identify

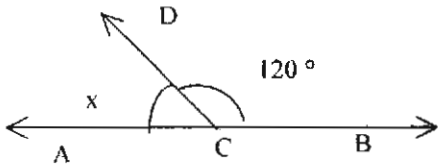
(a) two pairs of adjacent angles on a line

(b) straight angle

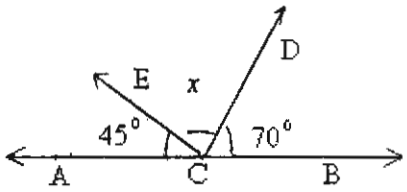


2. Find the values of x in the following figures .

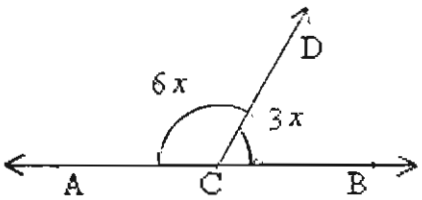
(a)



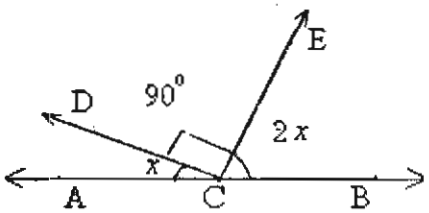
(b)



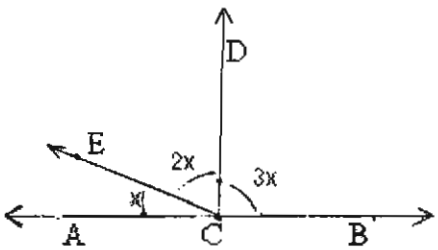
(c)



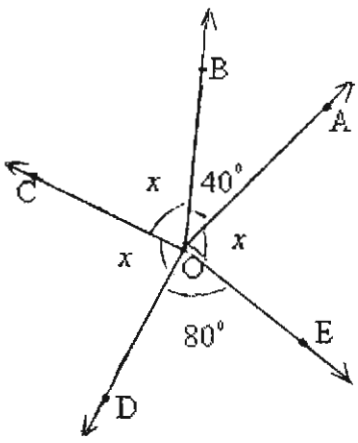
(d)



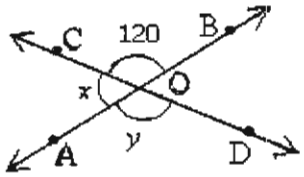
(e)



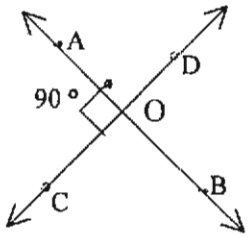
(f)



3. In the following figure two lines AB and CD intersect at the point O. Find the values of x and y.



4. In the figure find the other three angles.



5.4.3 Parallel lines :

Two or more straight lines are said to be parallel to each other if they are in the same plane and do not intersect at any point when produced either side. In fig. 5.51, AB and CD are parallel lines and we write AB || CD.

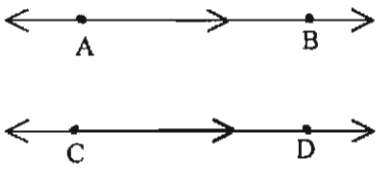


*Fig.*5.51

In our classroom we can find some examples for parallel lines.

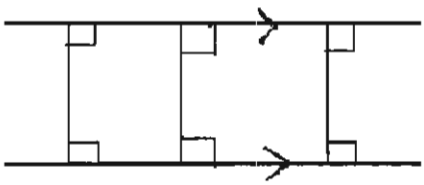


*Fig.*5.52

The opposite edges of the walls, the opposite edges of the doors, the opposite edges of a table top, the opposite edges of a blackboard are few examples for parallel lines. The rails on which the train moves is a very good example for parallel lines.

The perpendicular distance between the two parallel lines is the same everywhere. (*Fig.*5.52)

5. 4. 4 Transversal :

A straight line that intersects two or more parallel lines is called a transversal.

AB and CD are two parallel lines. PQ is the transversal that cuts AB at L and CD at M. (Fig. 5.53)

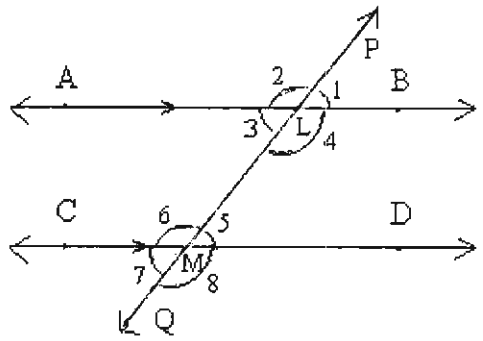


Fig. 5.53

a. Corresponding angles :

In fig. 5.53 ,

$\angle 1$ and $\angle 5$ are corresponding angles ;

$\angle 2$ and $\angle 6$ are corresponding angles ;

$\angle 3$ and $\angle 7$ are corresponding angles ;

$\angle 4$ and $\angle 8$ are corresponding angles.

b. Alternate angles:

In fig. 5.53 ,

$\angle 3$ and $\angle 5$ are alternate angles ;

$\angle 4$ and $\angle 6$ are alternate angles.

c. Interior angles on the same side of the transversal :

In fig. 5.53 ,

$\angle 3$ and $\angle 6$ are interior angles ;

$\angle 4$ and $\angle 5$ are interior angles.

5.4.4 (a) Properties of parallel lines when they are cut by a transversal :

When two parallel lines are cut by a transversal,

(a) the corresponding angles are equal

(b) the alternate angles are equal

(c) the sum of the interior angles on the same side of the transversal is 180°.

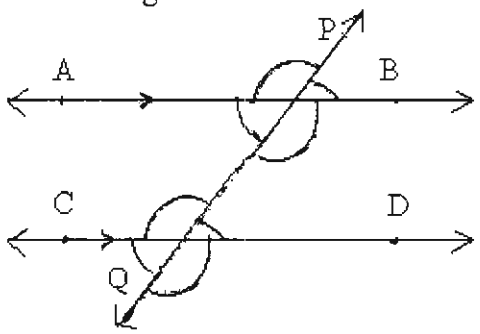


Fig. 5.54

In fig. 5.54 , AB and CD are parallel lines. PQ is a transversal. Since the corresponding angles are equal

$\angle 1 = \angle 5$; $\angle 2 = \angle 6$; $\angle 3 = \angle 7$; $\angle 4 = \angle 8$.

Since the alternate angles are equal

$\angle 3 = \angle 5$; $\angle 4 = \angle 6$.

Since the sum of the interior angles on the same side of the transversal is 180°

$\angle 3 + \angle 6 = 180°$;

$\angle 4 + \angle 5 = 180°$.

Activity : Fold a sheet of paper so as to get a pair of parallel lines. Again fold the paper across such that a transversal is obtained. Press the edges of the folded paper and open it. You wili see a pair of parallel lines with the transversal. Measure the angles and verify the properties of parallel lines when cut by a transversal.

Example 21:

In fig. 5.55 , identify and name the

(a) corresponding angles (b) alternate angles

(c) interior angles on the same side of the transversal

Solution:

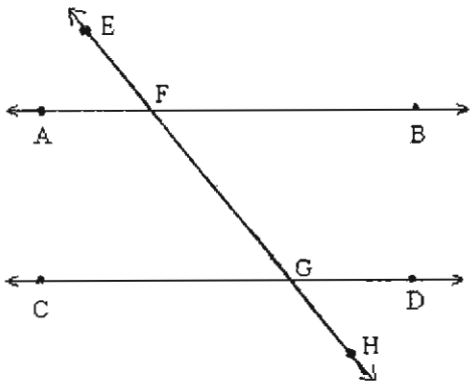


Fig. 5.55

(a) Corresponding angles :

1. $\angle EFB$ and $\angle FGD$
2. $\angle EFA$ and $\angle FGC$
3. $\angle BFG$ and $\angle DGH$
4. $\angle AFG$ and $\angle CGH$

(b) Alternate angles :

1. $\angle AFG$ and $\angle FGD$
2. $\angle BFG$ and $\angle FGC$

(c) Interior angles on the same side of the transversal :

1. $\angle AFG$ and $\angle FGC$
2. $\angle BFG$ and $\angle FGD$

Example 22 :

In fig. 5.56 , if $\angle XPB = 135°$ find a) $\angle PQD$ b) $\angle XPA$ c) $\angle PQC$

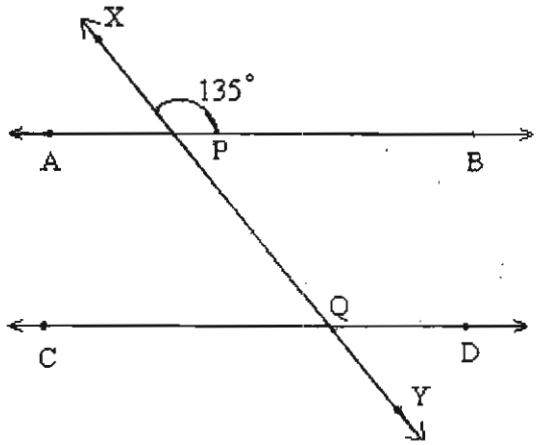


Fig. 5.56

Solution :

(a) $\angle XPB = 135°$ (given)

$\angle PQD = 135°$ (since $\angle XPB$ and $\angle PQD$ are corresponding angles)

(b) $\angle XPA = 180 - \angle XPB$ (since $\angle XPA$ and $\angle XPB$ are adjacent angles)

$= 180 - 135$

$= 45$

$\therefore \angle XPA = 45°$

(c) $\angle PQC = \angle XPA$

$= 45$ (since $\angle XPA$ and $\angle PQC$ are corresponding angles)

$\therefore \angle PQC = 45°$

Example 23 :

In fig.5.57 , if $\angle AFG = 100°$ find (a) $\angle FGD$ (b) $\angle BFG$ (c) $\angle FGC$

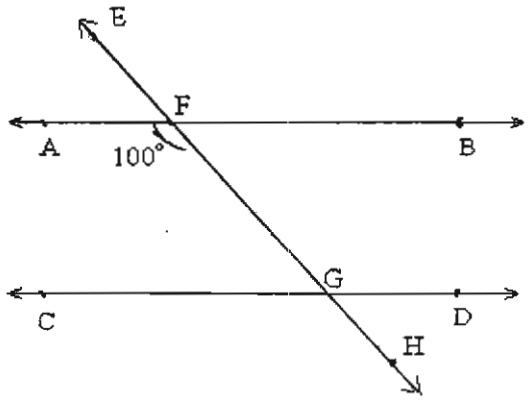


Fig.5.57

Solution :

(a) We know that alternate angles are equal

Here $\angle AFG$ and $\angle FGD$ are alternate angles.

$\therefore \angle AFG = \angle FGD = 100°$

$\therefore \angle FGD = 100°$

(b) $\angle BFG = 180 - \angle AFG$ (since $\angle AFG$ and $\angle BFG$ are adjacent angles)

$= 180 - 100$

$= 80$

$\therefore \angle BFG = 80°$

(c) We know that the sum of the interior angles on the same side of the transversal is 180°.

$\therefore \angle AFG + \angle FGC = 180°$

$100 + \angle FGC = 180$

$\angle FGC = 180 - 100$

$= 80$

$\therefore \angle FGC = 80°$

Example 24:

In fig.5.58, find x and y .

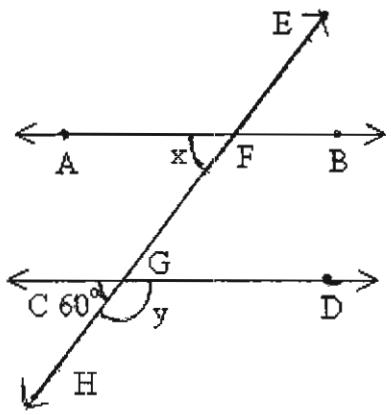


Fig. 5.58

Solution :

$\angle CGH = 60°$ (given)

$\angle CGH = \angle AFG$

$\angle AFG = 60°$ (since $\angle CGH$ and $\angle AFG$ are corresponding angles)

$\therefore x = 60°$

$\angle CGH + \angle DGH = 180°$ (since sum of the two adjacent angles =180°)

$60 + \angle DGH = 180$

$\angle DGH = 180 - 60$

$= 120$

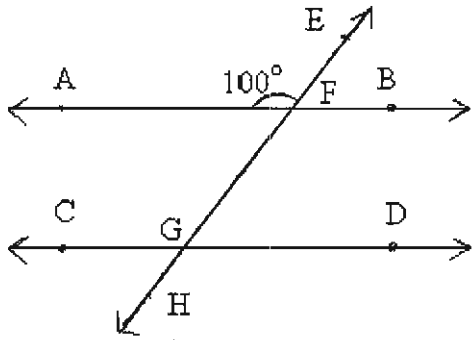
$\angle DGH = 120°$

$\therefore y = 120°$

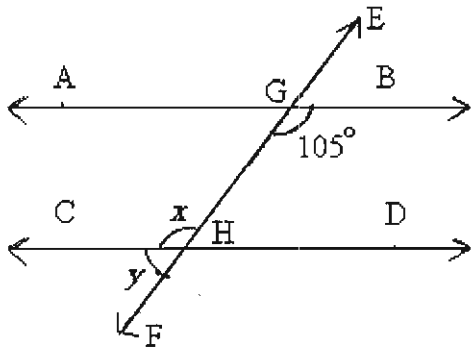
Exercise5.6

1.In figure $\angle EFA = 100°$. *Find*

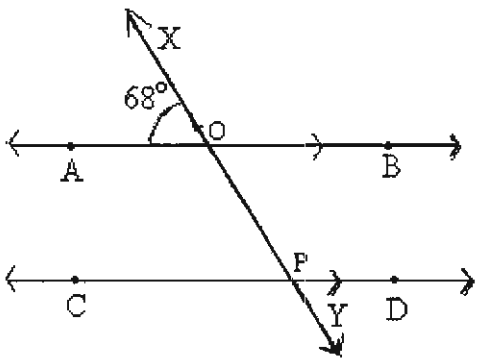
a) $\angle FGC$ *b)* $\angle EFB$ *c)* $\angle FGD$



2.Find the values of x and y from the figure if $\angle BGH = 105°$



3.In figure AB || CD and the transversal XY cuts AB and CD at O and P respectively If $\angle XOA = 68°$, *find the remaining angles formed at O and at P*



5.5 Triangles :

5.5.1 Sum of the angles of a triangle:

In the figures given below , measure the three angles of each triangle and tabulate them. Also find the sum of the three angles of each triangle.

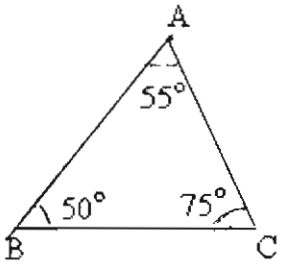


Fig. 5.59(a)

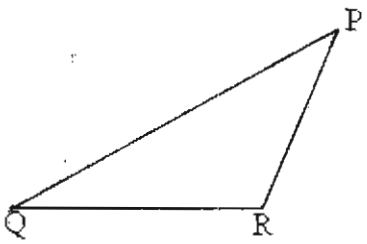


Fig. 5.59(b)

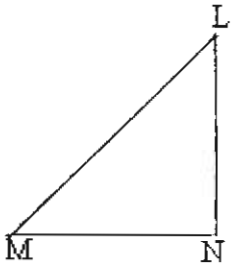


Fig. 5.59(c)

Figure	The angles of triangle			Total
5.59 (a)	$\angle A = 55°$	$\angle B = 50°$	$\angle C = 75°$	180°
5.59(b)	$\angle P =$	$\angle Q =$	$\angle R =$	
5.59(c)	$\angle L =$	$\angle M =$	$\angle N =$	

Table. 5.1

What do you observe from the above results? In the above table the value under the total column is 180° for all the triangles. **Hence the sum of the three angles of a triangle is 180°.**

Activity: Draw any triangle ABC on a sheet of paper and mark the angles 1 , 2and 3 on both sides of the paper as shown in fig 5.60(a).

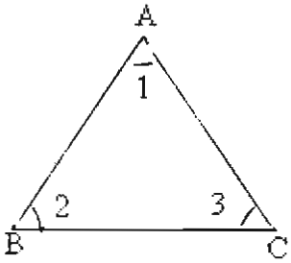


Fig. 5.60(a)

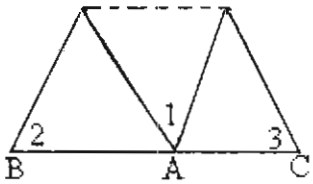


Fig. 5.60(b)

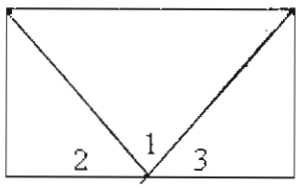


*Fig.*5.60(c)

Cut the triangle ABC. Fold the vertex A to touch the side BC as shown in the fig. 5.60(b). Fold the vertices B and C to get a rectangle as shown in fig. 5.60(c). Now we see that $\angle 1$, $\angle 2$ and $\angle 3$ make a straight angle.

Hence $\angle 1 + \angle 2 + \angle 3 = 180°$

(i.e) $\angle A + \angle B + \angle C = 180°$

∴ The sum of the three angles of a triangle is 180°

Example 25 :

Two angles of a triangle are 24° and 64°. Find the third angle.

Solution :

Let the third angle be x. we know that the sum of all three angles of a triangle is 180°.

$$24 + 64 + x = 180$$
$$88 + x = 180$$
$$x = 180 - 88$$
$$x = 92$$

∴ The third angles is 92°.

Example 26 :
In the fig. 5.61 find the angles of Δ ABC.
Solution :

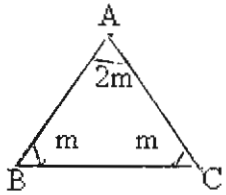


Fig. 5.61

We know that

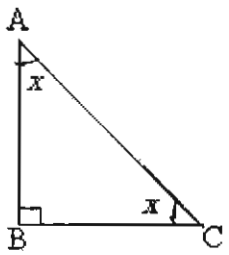
$$\angle A + \angle B + \angle C = 180°$$
$$\therefore\ 2m + m + m = 180$$
$$4m = 180$$
$$m = \frac{180}{4}$$
$$m = 45$$
$$\therefore \angle A = 2m = 2 \times 45 = 90°$$
$$\angle B = m = 45°$$
$$\angle C = m = 45°$$

THINGS TO REMEMBER

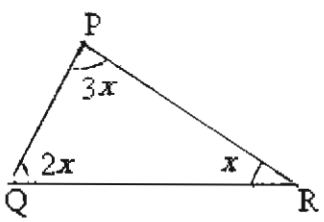
1. **The sum of the adjacent angles on a line is 180°.**
2. **Angle at a point is 360°.**
3. **If two straight lines intersect each other, the vertically opposite angles are equal.**
4. **Two or more straight lines are said to be parallel to each other if they are in the same plane and do not intersect at any point when produced either side.**
5. **A straight line that intersects two or more parallel lines is called a transversal.**
6. **When a transversal intersects two parallel lines then**
   (a) the corresponding angles are equal,
   (b) the alternate angles are equal,
   (c) the sum of the interior angles on the same side of a transversal is 180°.
7. **Sum of the angles of a triangle is 180°.**

Exercise 5.7

1.In the triangle ABC , find the value of x.

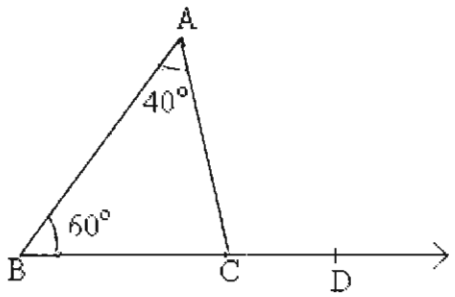


2. In the figure find the angles of Δ PQR.



3. If one of the two equal angles of an isosceles Δ is 50°, find the other two angles.

4. In the triangle ABC two angles A and B are given. What is the measure of $\angle ACB$ *and* $\angle ACD$ *?*



5.In a triangle all the angles are equal. Find each angle.

UNIT TEST - 5

Part - I *(15 ×1 = 15)*

Choose the correct answer :

1. The number of lines of symmetry in a square is

a) 4 *b) 2* *c) 6* *d) 0*

2. The order of rotational symmetry of an equilateral triangle is

a) 4 *b) 6* *c) 0* *d) 3*

3. Every translation has

a) a direction *b) a distance* *c) a direction and a distance*

d) a time

4. The reflection of Z is

a) Z *b)* Σ *c)* И *d)* N

5. The total angle formed at a point is

a) 90° *b) 180°* *c) 270°* *d) 360°*

6. The sum of adjacent angles on a line is

a) 90° *b) 180°* *c) 360°* *d) 0°*

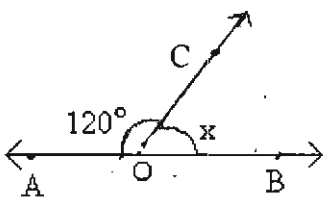
Fill in the blanks :

7. *The line of symmetry of any line segment is the _____ of the line.*
8. *Shapes or objects having one or more lines of symmetry are said to have ___ symmetry.*
9. *A line of symmetry divides the figure into ___ equal halves.*
10. *The order of rotational symmetry of a hexagon is ____.*
11. *Objects having rotational symmetry of order one is said to have ____.*
12. *The figures having no lines of symmetry can have _____ symmetry.*
13. *A parallelogram has ___ line of symmetry.*
14. *If two straight lines intersect each other, then the vertically opposite angles are ____.*
15. *When two parallel lines are cut by a transversal , the sum of the interior angles on the same side of the transversal is _____ degrees.*

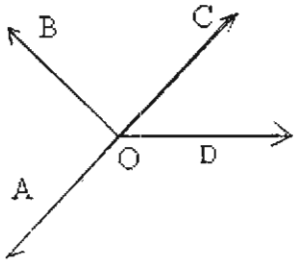
Part II *(10 × 2 = 20)*

Answer in one or two steps:

16. *What are the two types of symmetry?*
17. *Name any three letters of the English alphabets which have symmetry about the vertical axis.*
18. *How many sides do (a) a pentagon (b) a hexagon have?*
19. *What are the shapes that are used for tessellation?*
20. *What is meant by transformation?*
21. *What is meant by direct isometric transformation?*
22. *What will be the reflection of* D *?*
23. *From the figure find the value of x.*



24. From the figure write two pairs of adjacent angles.



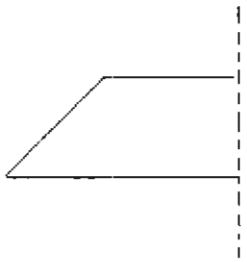
25. Write any two properties of parallel lines when they are cut by a transversal.

Part III ***(5 × 5 = 25)***

Give detailed answers :

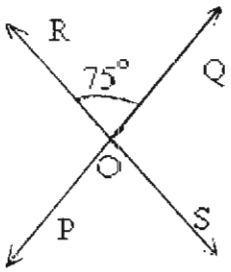
26. Draw all the lines of symmetry in a square.

27. Draw another part of the figure with respect to the given line of symmetry.

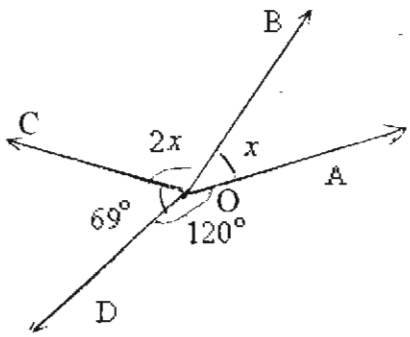


28. What is meant by rotation? Give an example.

29. PQ and RS are two straight lines intersect at O. If $\angle ROQ = 75°$, find the other three angles.



30. Find $\angle BOC$ and $\angle AOB$ in the figure.



6. PRACTICAL GEOMETRY

Practical geometry helps us to develop the skill of drawing figures for given measurements. The concepts learnt by us in theoretical geometry are put to application in practical geometry. If we remember the properties of the figures learnt in theoretical geometry, we can draw the figures easily and accurately in practical geometry. In this class let us learn the construction of parallel lines, triangles, division of a line segment and enlargement of triangles.

6.1 Parallel lines
6.2 Construction of triangles
6.3 Division of a line segment
6.4 Similarity

6.1 Parallel lines:

6.1.1 Construction of a line parallel to a given line through a point outside the given line using a set square and a ruler.

In theoretical geometry we have learnt about parallel lines. Let us recall what are parallel lines. Two or more straight lines are said to be parallel to each other if they are in the same plane and do not intersect at any point when produced either side. In this chapter let us learn how to construct a line parallel to a given line through a given point using a set square and a ruler.

6.1.1 Construction of a line parallel to a given line through a point outside the given line using a set square and a ruler.

Example 1:
Draw a line AB. Mark a point P above the line AB. Draw a line through P parallel to the line AB using a set square and a ruler.

Construction :

Step 1: Draw the line AB. (Fig. 6.1)

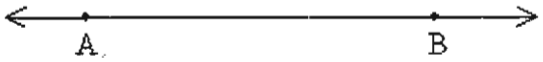


Fig. 6.1

Step 2 : Mark a point P above the line AB. (Fig. 6.2)

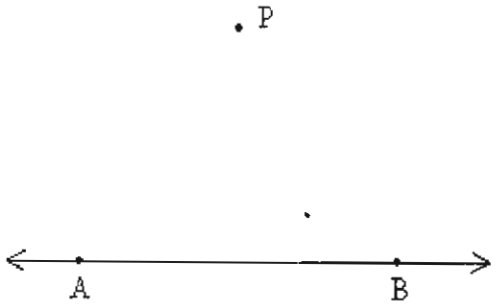


Fig. 6.2

Step 3: Place one edge of a set square containing the right angle along the given line AB and place

the ruler along the other edge as shown in fig. 6.3.

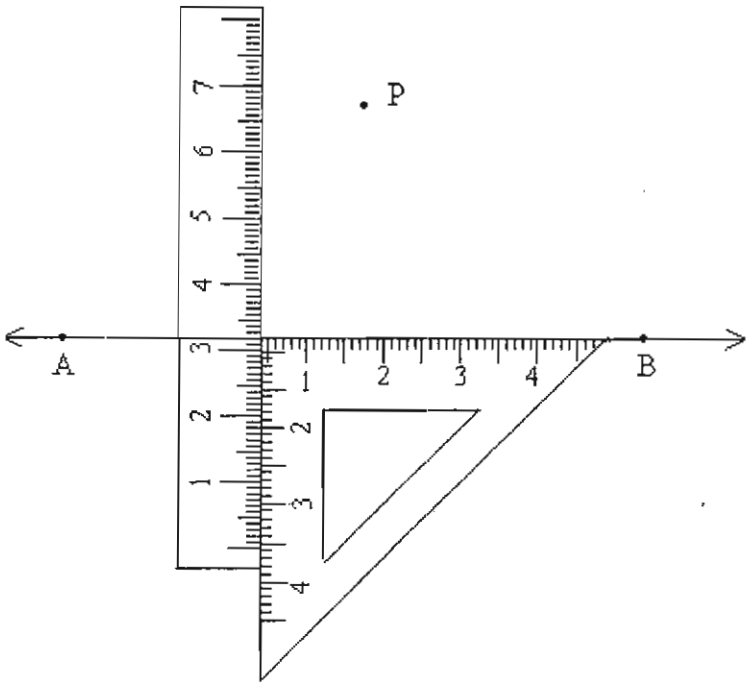


Fig. 6.3

Step 4: Pressing tightly the ruler with the left hand , slide the set square up along the ruler till the edge of the set square touches the point P as shown in fig.6.4.

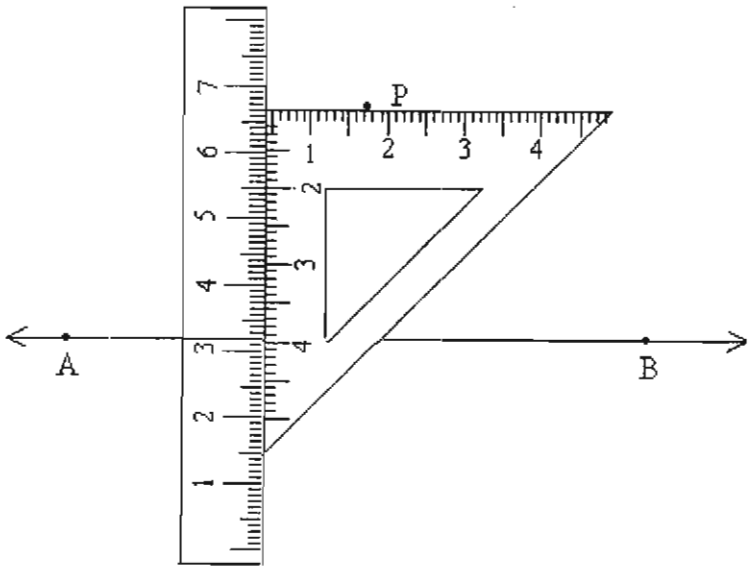


Fig. 6.4

Step 5: Through P draw a line along the edge. This is the required line through P and parallel to the line AB. (Fig. 6.5)

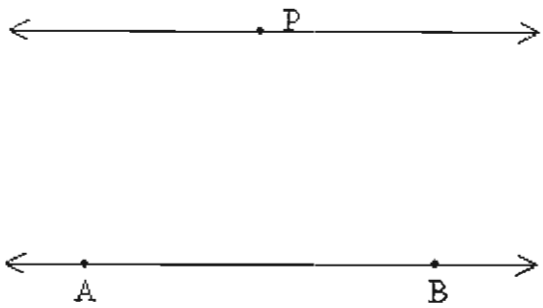


Fig. 6.5

Note : If the point P is below the line AB slide the set square down along the ruler till the edge of the set square touches the point P.

Exercise 6.1

1. ***Draw a line AB . Mark a point P above the line AB . Draw a line through P and parallel to the line AB .***
2. ***Draw a line PQ . Mark a point A below the line PQ . Draw a line through A and parallel to the line PQ .***
3. ***Draw a line AB . Mark two points P and Q such that P above the line AB and Q below the line AB . Draw lines through P and Q parallel to the line AB .***

6.2 Construction of Triangles :

6.2.1 Introduction
6.2.2 To construct a triangle when three sides are given
6.2.3 To construct an equilateral triangle
6.2.4 Right triangle
6.2.5 To construct a right triangle given the hypotenuse and one side
6.2.6 To construct a right triangle given two sides
6.2.7 To construct a triangle when two sides and one angle are given
6.2.8 To construct a triangle when two angles and one side are given

6.2.1 Introduction :

In the previous class we have learnt about the parts of a triangle and the various types of triangles in theoretical geometry. Let us recall here the types of triangles. Triangles can be classified into two groups one on the basis of sides and the other on the basis of angles.

Classification of triangles on the basis of sides :

a) If all the three sides of a triangle are not equal then it is called a **scalene triangle.**
b) If any two sides of a triangle are equal then it is called an **isosceles triangle.**
c) If all the three sides of a triangle are equal then it is called an **equilateral triangle.**

Classification of triangles on the basis of angles :

a) If each angle of a triangle is an acute angle , that is less than 90° then it is called an **acute angled triangle.**
b) In a triangle if any one angle is a right angle , that is equal to 90° then the triangle is called a **right angled triangle or right triangle.**
c) In a triangle if any one angle is an obtuse angle , that is greater than 90° then the triangle is called an **obtuse angled triangle.**

In this chapter let us learn how to construct different types of triangles. To construct a triangle we need three measurements in which atleast the length of one side must be given.

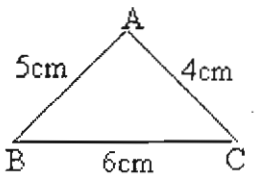
6.2.2 To construct a triangle when three sides are given :

Example 2 :

Construct a triangle ABC such that AB = 5 cm, BC = 6 cm and AC = 4 cm.

Construction :

Draw a rough figure and mark the given measures.



Rough Figure

Fig 6.6

Step 1: Draw the line segment BC = 6 cm. (Fig. 6.7)

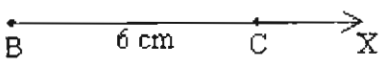
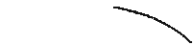


Fig. 6.7

Step 2: With centre B and radius 5 cm (= AB) draw an arc of the circle. (Fig. 6.8)



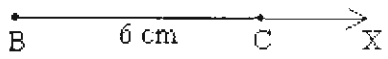
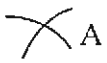


Fig. 6.8

Step 3: With centre C and radius 4 cm (= AC) draw another arc intersecting the first arc at A. (Fig. 6.9)



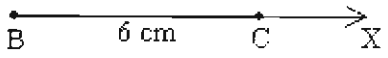


Fig. 6.9

Step 4: Join A and B. Join A and C. Then Δ ABC is the required triangle. (Fig. 6.10)

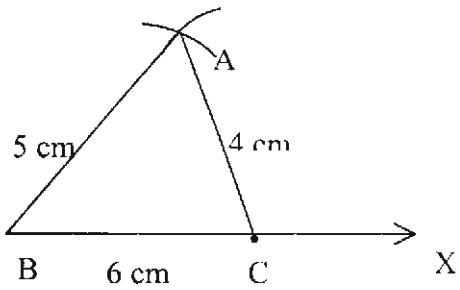
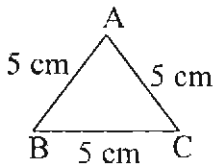


Fig. 6.10

6.2.3 To construct an equilateral triangle :

Example 3 :
Construct an equilateral triangle of side 5 cm.



Rough figure
Fig. 6.11

Construction :

Draw a rough figure and mark the measures.

Step 1: Draw the line segment BC = 5 cm. (Fig. 6.12)

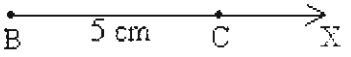


Fig. 6.12

Step 2: With B as centre and radius 5 cm (= AB) draw an arc of the circle.(Fig. 6.13)

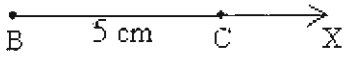


Fig. 6.13

Step 3: With C as centre and radius 5 cm (= AC) draw another arc intersecting the first arc at A. (Fig. 6.14)

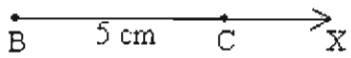


Fig. 6.14

Step 4: Join A and B. Join A and C. Then Δ ABC is the required triangle. (Fig. 6.15)

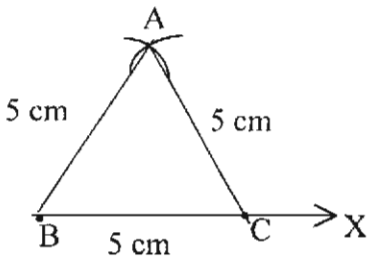


Fig. 6.15

6.2.4 Right triangle :

In fig. 6.16 triangle ABC is a right triangle. $\underline{|B}$ is the right angle or the triangle is right angled at B. In a right triangle the side opposite to the right angle is called the hypotenuse of the triangle. AC is the hypotenuse and AB and BC are the sides of the triangle ABC.

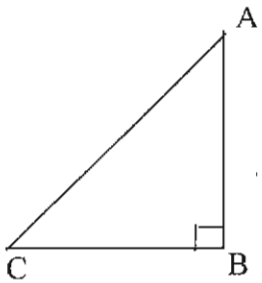
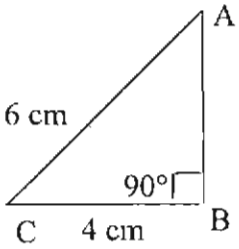


Fig. 6.16

6.2.5 To construct a right triangle given the hypotenuse and one side :

Example 4 :
Construct a right triangle with hypotenuse 6 cm and one side 4 cm.



Rough figure
Fig. 6.17

Construction :
Draw a rough figure and mark the measures.
Step1: Draw a line segment CB of length 4 cm. (Fig. 6.18)

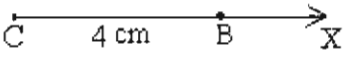


Fig. 6.18

Step 2: Draw $\angle CBY$ of measure 90°. (Fig. 6.19)

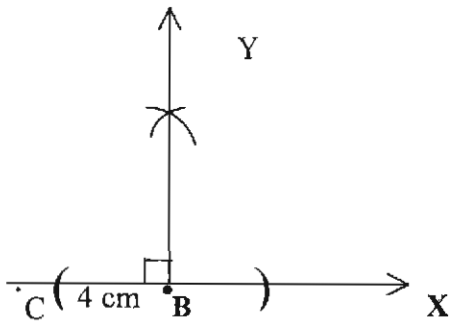


Fig. 6.19

Step 3: With C as centre radius 6 cm (= hypotenuse) draw an arc of the circle to intersect BY at A. (Fig. 6.20)

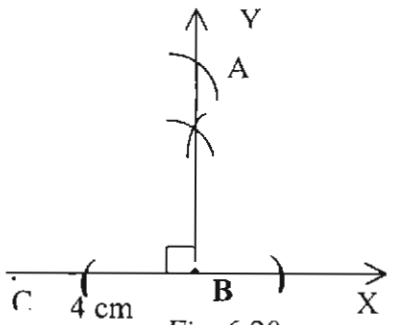


Fig. 6.20

Step 4: Join C and A. Now triangle ABC is the required right triangle. (Fig. 6.21)

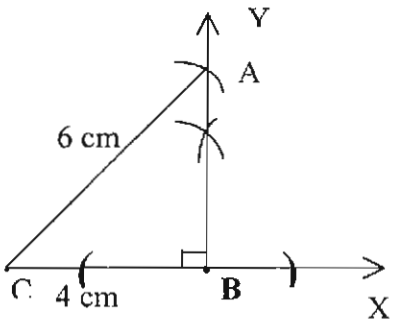


Fig. 6.21

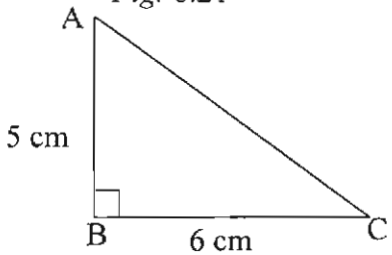
6.2.6 To construct a right triangle given two sides :

Example 5 :

Construct a right triangle ABC with AB = 5 cm, BC = 6 cm and $\angle B = 90°$

Construction :

Draw a rough figure and mark the measures.



Rough figure

Fig. 6.22

Step 1: Draw a line segment BC of length 6 cm. (Fig. 6.23)

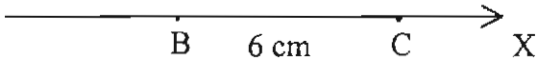


Fig. 6.23

Step 2: Draw $\angle CBY$ of measure 90°. (Fig. 6.24)

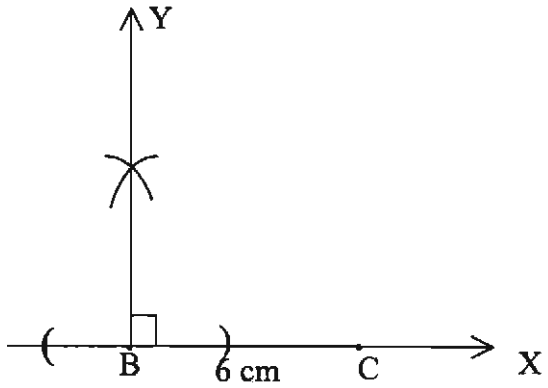


Fig. 6.24

Step 3: With B as centre and radius 5 cm draw an arc of the circle to intersect BY at A (Fig 6.25)

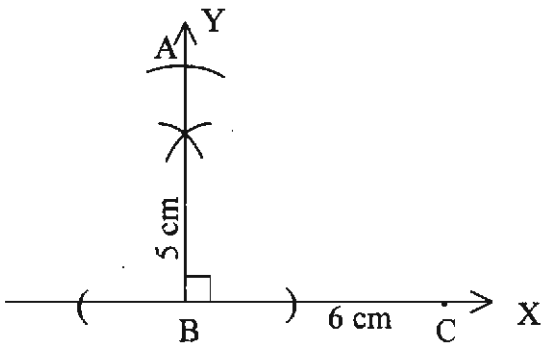


Fig. 6.25

Step 4: Join A and C. Now triangle ABC is the required right triangle. (Fig. 6.26)

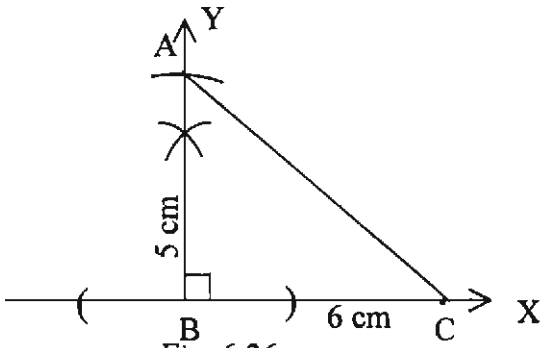


Fig. 6.26

Discuss: To construct a triangle we need three measurements. But to construct a right triangle it is enough to have only two measurements. Why?.

Exercise 6.2

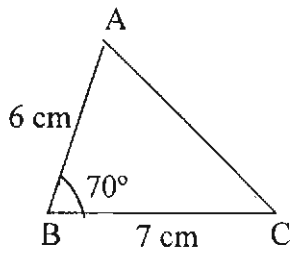
1. ***Construct a triangle ABC given AB = 4 cm, BC = 7 cm and AC = 5 cm.***
2. ***Construct a triangle PQR given PQ = 5.5 cm, QR = 6.5 cm and PR = 3.5 cm.***
3. ***Construct a triangle XYZ given XY = 6 cm , YZ = 8 cm and XZ = 7 cm.***
4. ***Construct an equilateral triangle ABC with side 6 cm.***

5. ***Construct an equilateral triangle PQR with PQ = QR = RP = 7 cm.***
6. ***Construct an equilateral triangle XYZ with each side 7.5 cm.***
7. ***Draw a right triangle with hypotenuse 7 cm and one side 5 cm. Measure the other side.***
8. ***Draw a right triangle with hypotenuse 5 cm and one side 4 cm. Measure the other side and other two angles.***
9. ***Construct a right triangle ABC in which $\angle C = 90^\circ$, AB = 8 cm and BC = 6 cm. Measure AC and other two angles.***
10. ***Draw a right triangle ABC in which AB = 6 cm, BC = 7 cm and $\angle B = 90^\circ$ Measure the hypotenuse and other two angles.***
11. ***Draw a right triangle PQR in which PR = RQ = 8 cm and $\angle R = 90^\circ$. Measure the hypotenuse and other two angles.***
12. ***Draw a right triangle ABC in which AC = 9 cm, CB = 8 cm and $\angle C = 90^\circ$. Measure the hypotenuse and other two angles.***

6.2.7 To construct a triangle when two sides and one angle are given :

Example 6 :
Construct a triangle ABC given AB = 6 cm, BC = 7 cm and $\angle B = 70^\circ$.

Construction :
Draw a rough figure and mark the measures.



Rough figure
Fig. 6.27

Step 1: Draw a line segment BC = 7 cm. (Fig. 6.28)

Fig. 6.28

Step 2: Draw $\angle XBY = 70^\circ$. (Fig. 6.29)

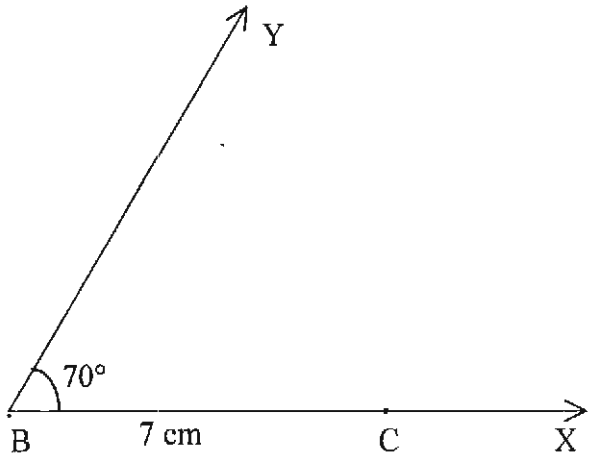


Fig. 6.29

Step 3: With B as centre and radius 6 cm (= AB) cut an arc on BY and mark the point as A. (Fig. 6.30)

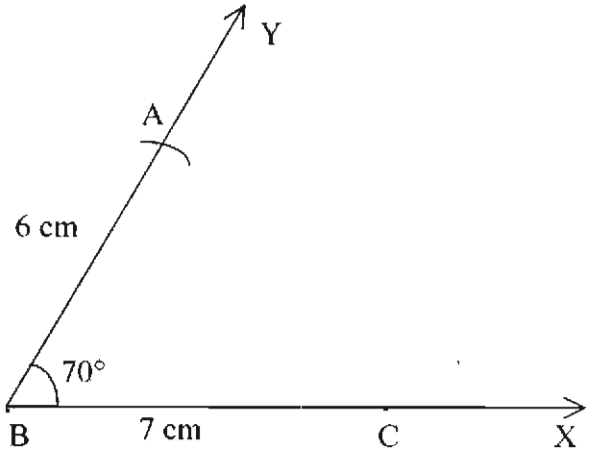


Fig. 6.30

Step 4: Join A and C. Then triangle ABC is the required triangle. (Fig. 6.31)

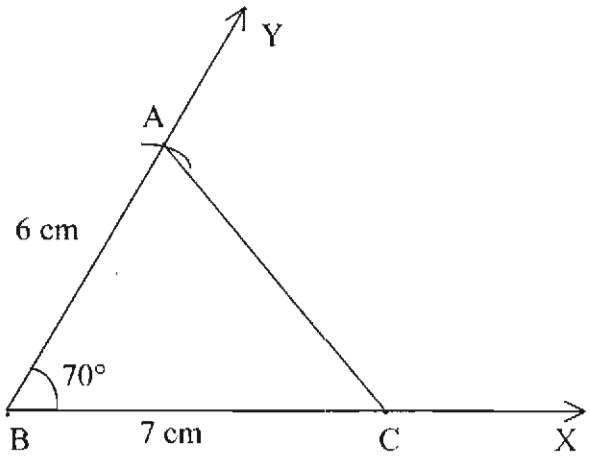
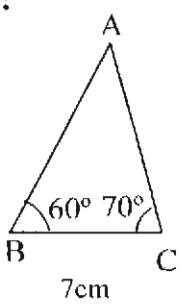


Fig. 6.31

6.2.8 To construct a triangle when two angles and one side are given :

Example 7 :
Construct a triangle ABC given BC = 7 cm ,
$\underline{|B} = 60°$ and $\underline{|C} = 70°$.

Construction :
Draw a rough figure and mark the measures.



Rough Figure
Fig. 6.32

Step 1: Draw a line segment BC of length 7 cm. (Fig. 6.33)

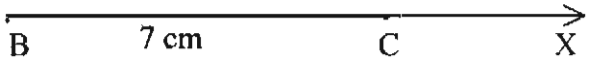


Fig. 6.33

Step 2: Draw BY such that $\angle XBY = 60°$. (Fig. 6.34)

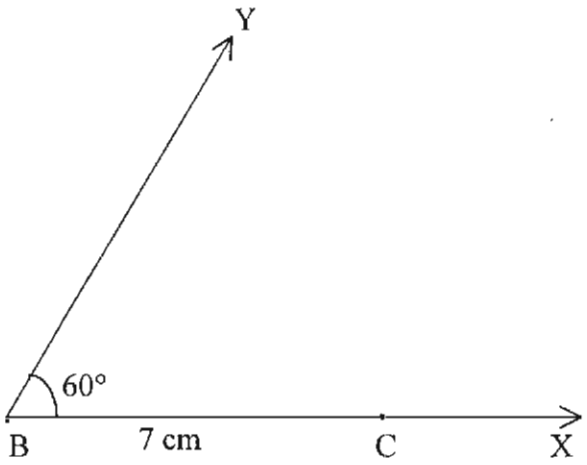


Fig. 6.34

Step 3: Draw CZ such that $\angle BCZ = 70°$. (Fig. 6.35)

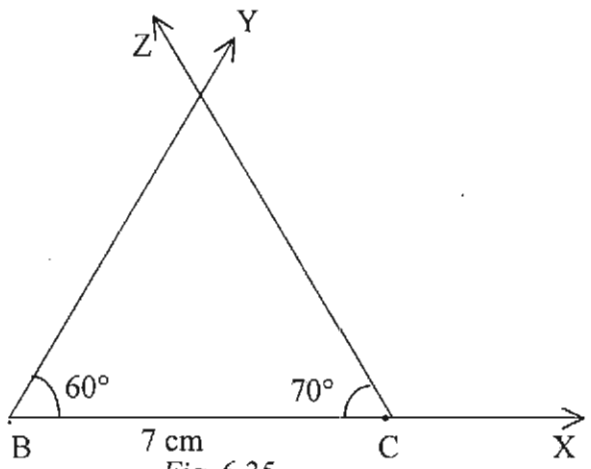


Fig. 6.35

Step 4: BY and CZ intersect at A. Then triangle ABC is the required triangle. (Fig. 6.36)

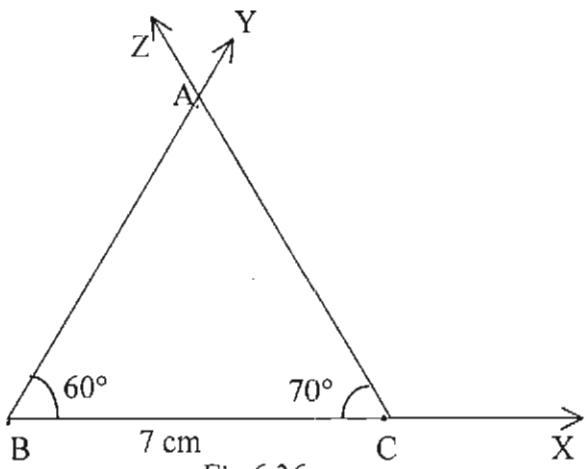


*Fig.*6.36

Exercise 6.3

1. *Construct a triangle ABC given AB = 5 cm, BC = 6 cm and* $\angle B = 50°$.
2. *Construct a triangle PQR with PQ = 7 cm, QR = 8 cm and* $\angle PQR = 80°$.
3. *Construct a triangle XYZ given XY = 7 cm,* $\angle XYZ = 60°$ *and YZ = 7 cm. Measure* $\angle X$ *and* $\angle Z$. *What do you observe?*
4. *Construct a triangle ABC given BC = 6 cm,* $\angle B = 50°$ *and* $\angle C = 60°$.
5. *Construct a triangle PQR with QR = 8 cm,* $\angle Q = 80°$ *and* $\angle R = 80°$. *Measure the sides PQ and PR. What do you find?*
6. *Construct a triangle XYZ given XY = 6 cm,* $\angle X = 55°$ *and* $\angle Y = 65°$.

THINGS TO REMEMBER

1. **For the construction of a triangle, rough figure should be drawn and the given measures should be marked.**
2. **Rough figure need not be drawn with exact measures.**
3. **At least one side of the triangle must be given to construct a triangle.**
4. **If only one side is given, that side must be taken as the base of the triangle.**
5. **If three sides are given the longest side must be taken as the base of the triangle.**
6. **A triangle can be drawn if the measurements are given in the following order.**
   - **(a) Side – Side – Side (SSS)**
   - **(b) A Side of an equilateral triangle (SSS)**
   - **(c) Hypotenuse and one side (RHS)**
   - **(d) Sides containing a right angle (SAS)**
   - **(e) Two sides and one angle (SAS)**
   - **(f) Two angles and the included side (ASA)**

6.3 Division of a line segment:

In the previous class we have learnt to construct a line segment of a given measurement. We now learn how to divide a given line segment into equal parts.

6.3.1 To divide a given line segment into equal parts

6.3.2 To divide a given line segment in a given ratio

6.3.1 To divide a given line segment into equal parts :

Example 8 :
Draw a line segment AB of length 6 cm and divide it into 3 equal parts.

Construction:
Step 1: Draw the line segment AB = 6 cm. (Fig. 6.37)

*Fig.*6.37

Step 2: At A, draw any ray AY making an acute angle with AB. (Fig. 6.38)

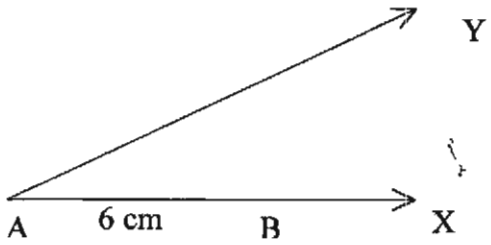


Fig. 6.38

Step 3: On AY cut off 3 equal segments AP, PQ, QR using the compass. (Fig. 6.39)

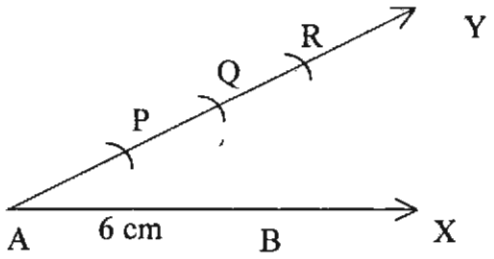


Fig. 6.39

Step 4: Join R and B. (Fig. 6.40)

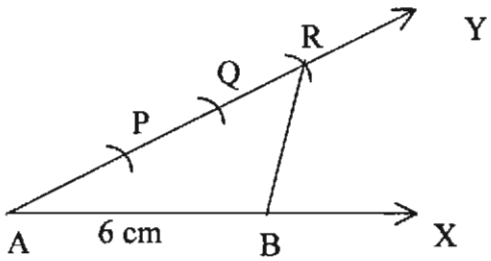


Fig. 6.40

Step 5: Through Q draw a line parallel to RB using set squares to cut AB in D. Through P draw a line parallel to RB to cut AB in C. AC , CD , DB are the 3 equal parts of AB. Here AB is divided into 3 equal parts. (Fig. 6.41)

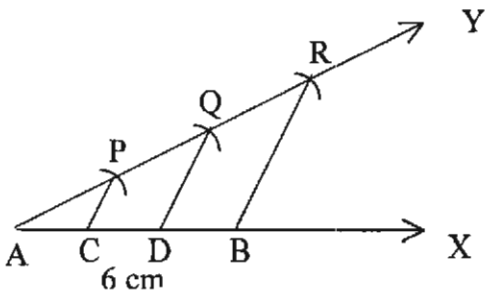


Fig. 6.41

Note: Measure the length of each line segment AC, CD and DB. Verify whether they are equal.

6.3.2 To divide a given line segment in a given ratio :

Example 9 :

Draw a line segment PQ of length 10 cm. Divide the line segment in the ratio 3:2.

Construction:

Step 1: Draw the line segment PQ = 10 cm. (Fig. 6.42)

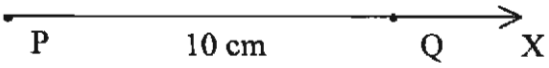


Fig. 6.42

Step 2: At P, draw any ray PY making an acute angle with PQ. (Fig. 6.43)

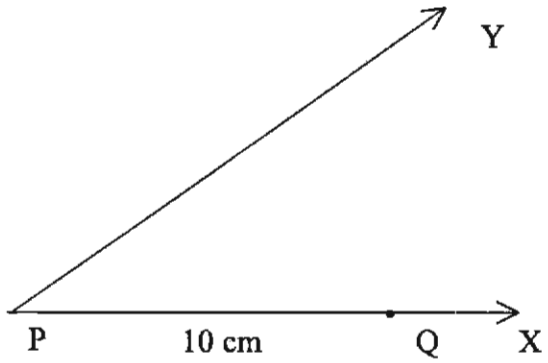


Fig. 6.43

Step 3: Since the ratio given is 3 : 2 we have to divide the line into 3 + 2 = 5 equal parts. Therefore on PY cut off 5 equal segments PA , AB , BC , CD and DE using the compass. (Fig. 6.44)

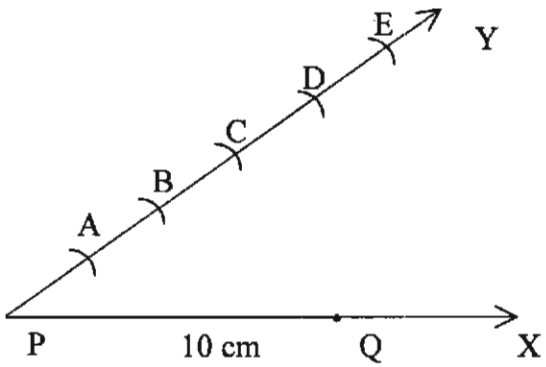


Fig. 6.44

Step 4: Join E and Q. (Fig. 6.45)

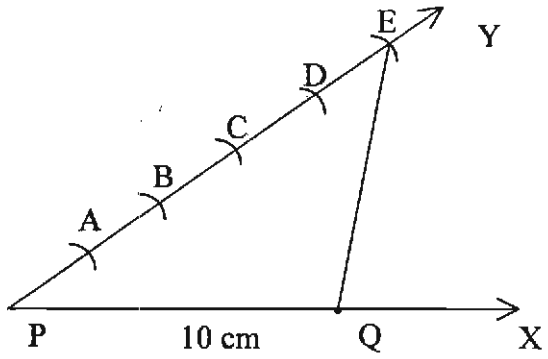


Fig. 6.45

Step 5: Since the ratio is 3 : 2 draw a line parallel to EQ through C which is at a distance of three equal parts from P to cut PQ at R.Now PQ is divided in the ratio 3 : 2 at R. (Fig 6.46) i.e. PR : RQ = 3 : 2.

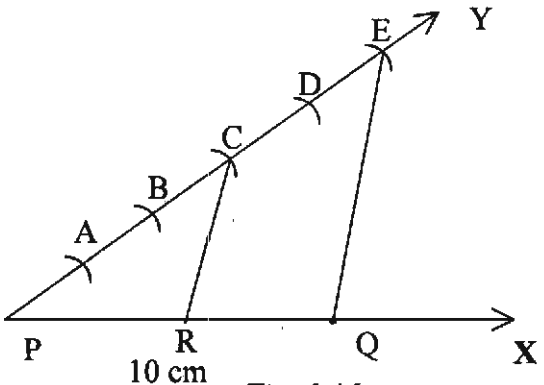


Fig. 6.46

Note: Measure the lengths of the line segments PR and RQ. Verify whether they are in the ratio 3 : 2.

Exercise 6.4

1. ***Draw a line segment AB of length 8 cm and divide it into 4 equal parts. Measure each part.***
2. ***Draw a line segment PQ of length 9 cm and divide it into 6 equal parts. Measure each part.***
3. ***Draw a line segment AB of length 7.5 cm and divide it into 5 equal parts. Measure each part.***
4. ***Draw a line segment AB of length 8 cm and divide it in the ratio 3 : 1***
5. ***Draw a line segment PQ of length 10 cm and divide it in the ratio 2 : 3***
6. ***Draw a line segment AB of length 9 cm and divide it in the ratio 3 : 2***

6.4 Similarity :

6.4.1 Enlargement
6.4.2 To enlarge a given triangle in the ratio 1 : K where K is a natural number.
6.4.3 To enlarge the given triangle by taking a point inside the triangle as center of similitude.

Consider the following squares.

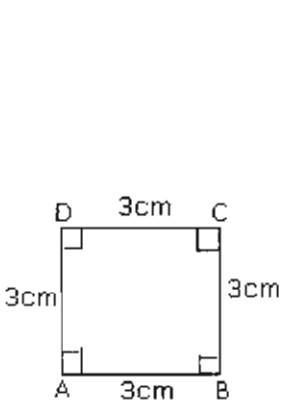


Fig. 6.47 (a)

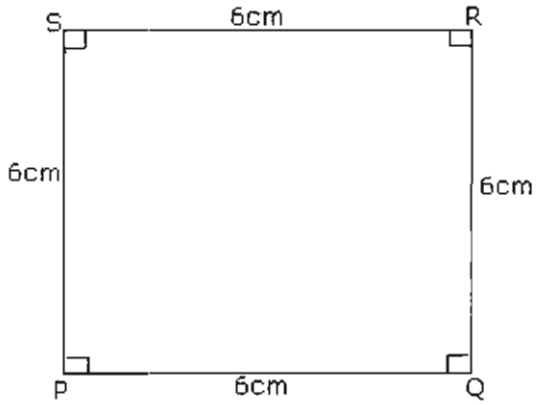


Fig. 6.47 (b)

In figures 6.47(a) and 6.47(b) ABCD and PQRS are two squares with side 3 cm and 6 cm respectively. The sides AB and PQ ; BC and QR ; CD and RS ; DA and SP are called corresponding sides.

Here $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SP} = \frac{1}{2}$$

$\angle A$ and $\angle P$; $\angle B$ and $\angle Q$; $\angle C$ and $\angle R$; $\angle D$ and $\angle S$ are called corresponding angles.

Here $\angle A = \angle P$
$\angle B = \angle Q$
$\angle C = \angle R$
$\angle D = \angle S$

Consider the following rectangles.

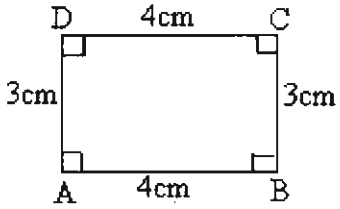


Fig. 6. 48(a)

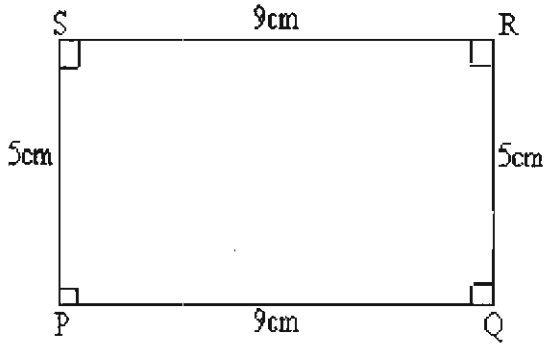


Fig. 6.48(b)

In figures 6.48 (a) and 6.48 (b) ABCD and PQRS are two rectangles. The length and breadth of the rectangle ABCD are 4 cm and 3 cm respectively. The length and breadth of the rectangle PQRS are 9 cm and 5 cm respectively. The sides AB and PQ ; BC and QR ; CD and RS ; DA and SP are called corresponding sides.

Here $\frac{AB}{PQ} = \frac{4}{9}$; $\frac{BC}{QR} = \frac{3}{5}$

$\frac{AB}{PQ} \neq \frac{BC}{QR}$

$\angle A$ and $\angle P$; $\angle B$ and $\angle Q$; $\angle C$ and $\angle R$; $\angle D$ and $\angle S$ are called corresponding angles.

Here $\angle A = \angle P$

$\angle B = \angle Q$

$\angle C = \angle R$

$\angle D = \angle S$

Consider the following triangles.

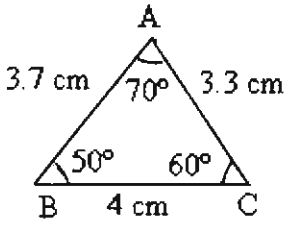


Fig. 6.49(a)

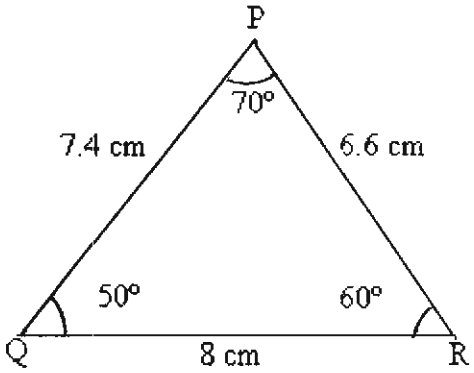


Fig. 6.49(b)

In figures 6.49 (a) and 6.49 (b) ABC and PQR are two triangles. The sides AB and PQ ; BC and QR ; CA and RP are called corresponding sides.

$$\text{Here } \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{1}{2}$$

$\angle A$ and $\angle P$; $\angle B$ and $\angle Q$; $\angle C$ and $\angle R$ are called corresponding angles.

Here $\angle A = \angle P$

$\angle B = \angle Q$

$\angle C = \angle R$

In the case of squares [Fig. 6.47 (a) and fig. 6.47(b)] the corresponding sides are proportional and the corresponding angles are equal. Hence the two squares of different sides are similar.

In the case of rectangles [Fig. 6.48(a) and fig. 6.48(b)] the corresponding angles are equal but the corresponding sides are not proportional. Hence these rectangles are not similar. Only those rectangles whose corresponding sides are proportional are similar.

In the case of triangles [Fig. 6.49(a) and fig. 6.49(b)] corresponding sides are proportional and corresponding angles are equal. Hence the two triangles ABC and PQR are similar.

In general **two plane figures are said to be similar if their corresponding sides are proportional and their corresponding angles are equal.**

For similarity in any other plane figures like parallelogram, quadrilateral both the properties should be satisfied.

Discuss : Can you say that figures of same shapes are similar?

6.4.1 Enlargement :

Enlargement means making larger the shape or the size of any object. For example a passport size photograph is an enlargement of a stamp size photograph. Enlarging a triangle means making larger the sides of the triangle in a given ratio satisfying the conditions for similarity. The original triangle and the enlarged triangle are of the same shape. All the sides of the enlarged triangle are larger than the corresponding sides of the original triangle in the given proportion. The point of intersection of the adjacent sides is called the centre of similitude. The enlargement is carried out with respect to the centre of similitude and the adjacent sides.

For enlargement of a triangle any two adjacent sides may be taken. For a triangle there are three centres of similitude since there are three vertices. We can enlarge the given triangle taking any one of the vertices as the centre of similitude. Enlargement can also be done by taking any point inside the triangle as the centre of similitude.

6.4.2 To enlarge a given triangle in the ratio 1:K where K is a natural number:

Example 10 :

Construct an equilateral triangle ABC of side 4 cm and enlarge it in the ratio 1 : 2.

Construction :

Step 1: Construct the equilateral triangle ABC of side 4 cm. (Fig. 6.50)

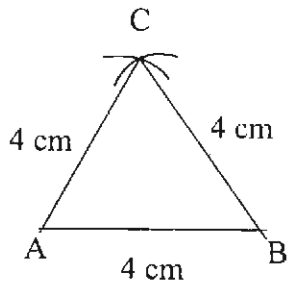


Fig. 6.50

Step 2: Taking A as the centre of similitude, draw the ray AX along AB and the ray AY along AC.(Fig. 6.51)

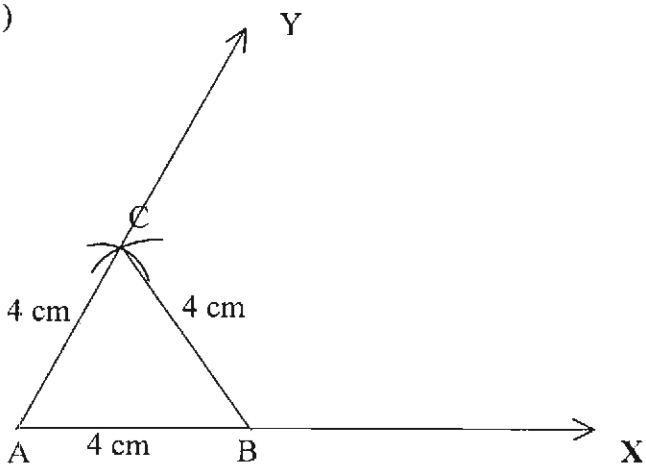


Fig. 6.51

Step 3 : Since the ratio of the enlargement is 1:2 the adjacent sides AB and AC of the original triangle are doubled in the enlarged triangle. Therefore with A as centre and radius 8 cm ($4 \times 2 = 8$) draw one arc cutting the ray AX at D and another arc cutting the ray AY at E using compass. (Fig. 6.52).

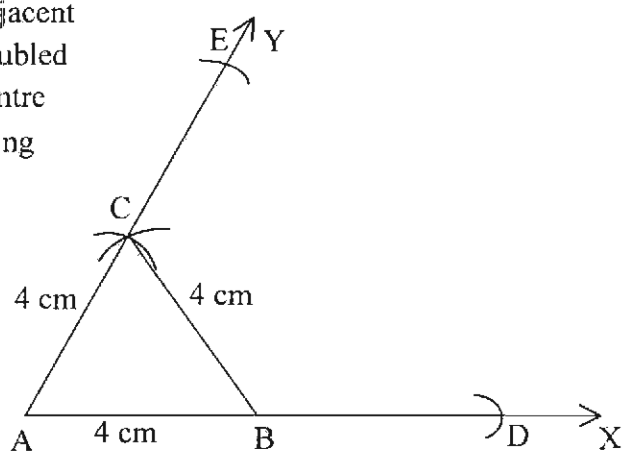


Fig . 6.52

Step 4: Join E and D (Fig 6.53) Now triangle ADE is the enlarged triangle of ABC in the ratio. 1 : 2.

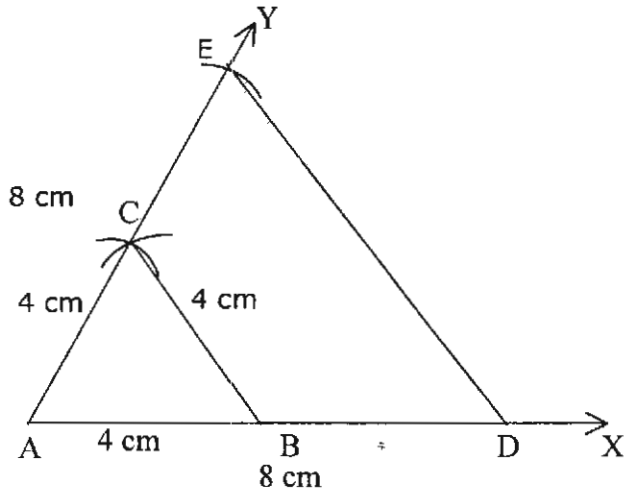


*Fig.*6.53

Example 11 :

Construct a right triangle ABC in which $\angle B = 90°$, BC = 3 cm , BA = 4 cm and enlarge it in the ratio 1 : 3.

Construction:

Step 1: Construct the right triangle ABC in which $\angle B = 90°$, BC = 3 cm and BA = 4 cm. (Fig. 6.54)

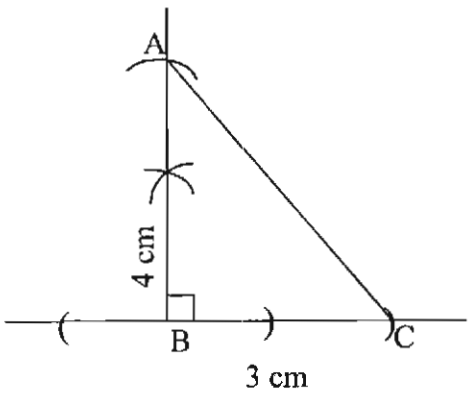


Fig. 6.54

Step 2: Taking B as the centre of similitude draw the ray BX along BC and the ray BY along BA. (Fig.6.55)

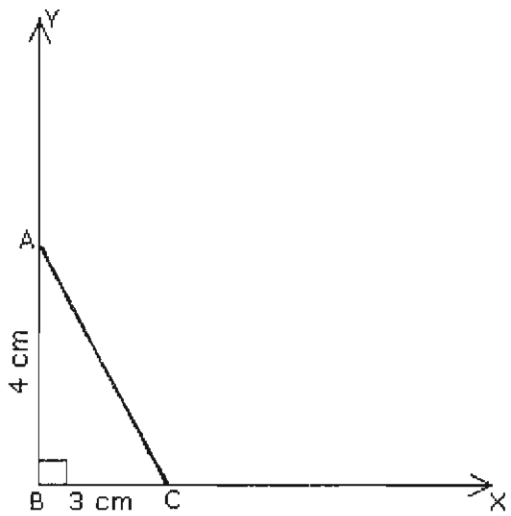


*Fig.*6.55

Step 3: With B as centre and radius 9 cm (3 × 3 =9)draw an arc cutting the ray BX at D using compass. (Fig. 6.56)

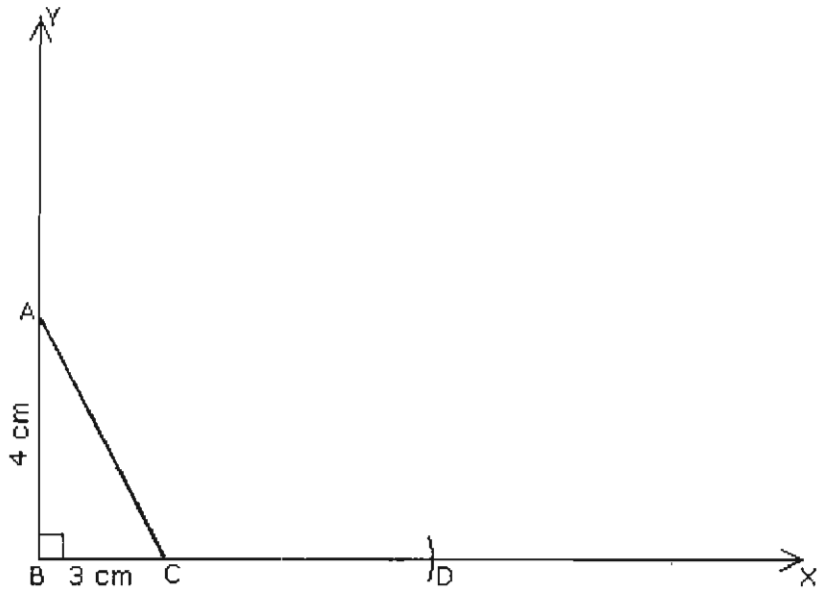


Fig. 6.56

Step 4: With B as centre and radius 12 cm (4 × 3 = 12) draw an arc cutting the ray BY at E using compass. (Fig. 6.57)

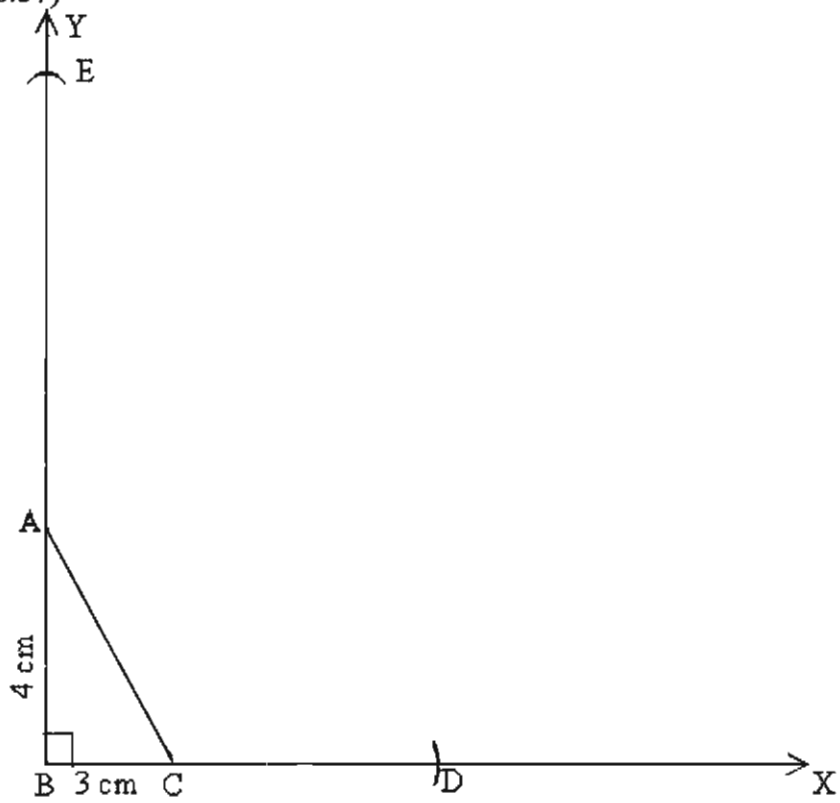


Fig. 6.57

Step 5: Join E and D. (Fig. 6.58) Now triangle EBD is the enlarged triangle of ABC in the ratio 1:3

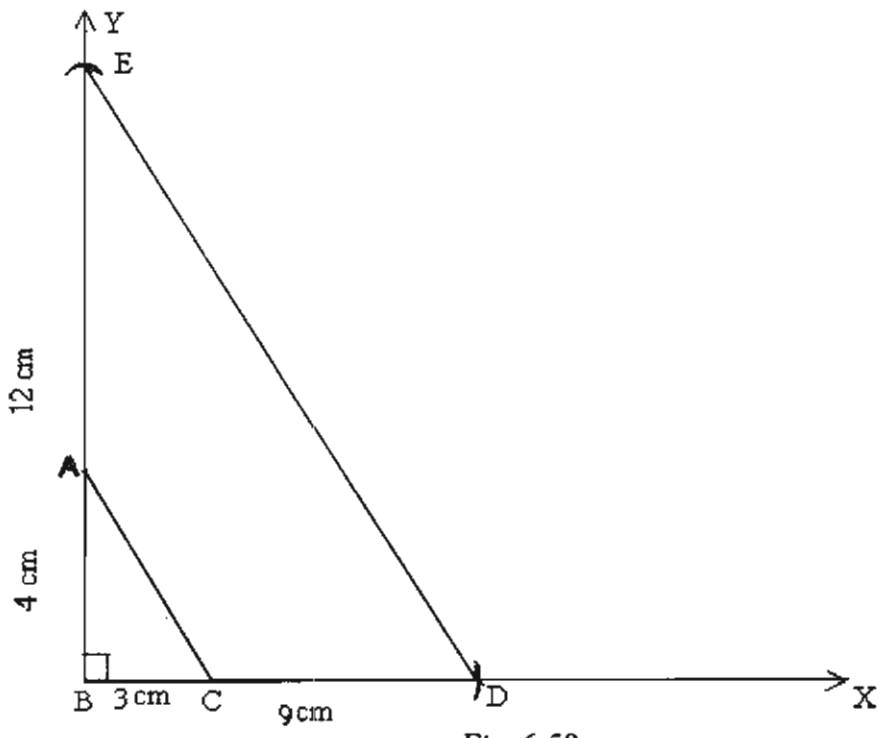


Fig. 6.58

6.4.3 To enlarge the given triangle by taking a point inside the triangle as centre of similitude :

Enlargement can be done by taking any point inside the original triangle as centre of similitude and the enlargement is made with respect to this point. In this case two new adjacent sides are drawn through tne centre of similitude parallel to the adjacent sides of the original triangle.

Example 12 :

Construct an isosceles triangle ABC in which AB = AC = 5 cm and BC = 4 cm. Enlarge the given triangle in the ratio 1 : 2 by taking any point inside the triangle as the centre of similitude.

Step 1: Construct the isosceles triangle in

which AB = AC = 5 cm and BC = 4 cm. (Fig 6.59)

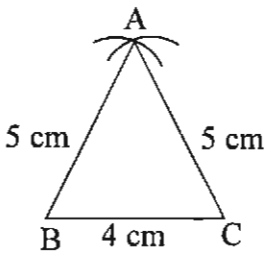


Fig. 6.59

Step 2: Take any point P inside the triangle ABC. (Fig. 6.60)

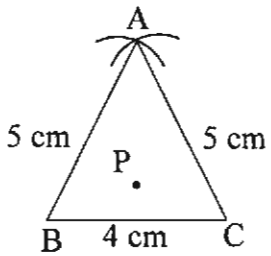


Fig. 6.60

Step 3: Through P draw a ray PX parallel to BC. (Fig. 6.61)

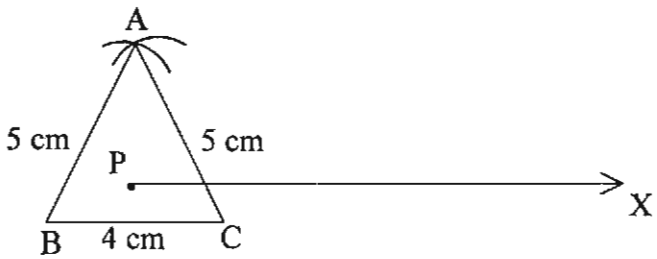


*Fig.*6. 61

Step 4: Through P draw a ray PY parallel to BA. (Fig. 6.62)

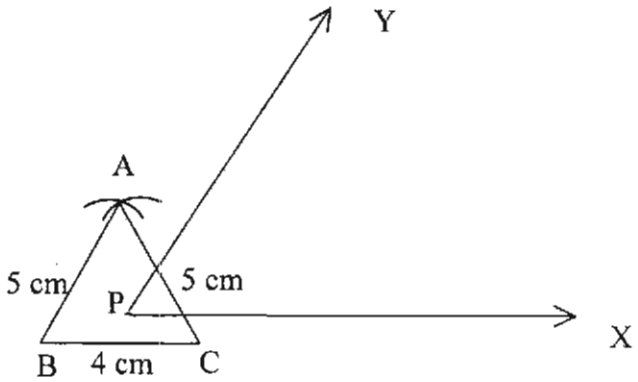


Fig. 6.62

Step 5: With P as centre and radius 8 cm ($4 \times 2 = 8$) draw an arc cutting the ray PX at D using compass. (Fig. 6.63).

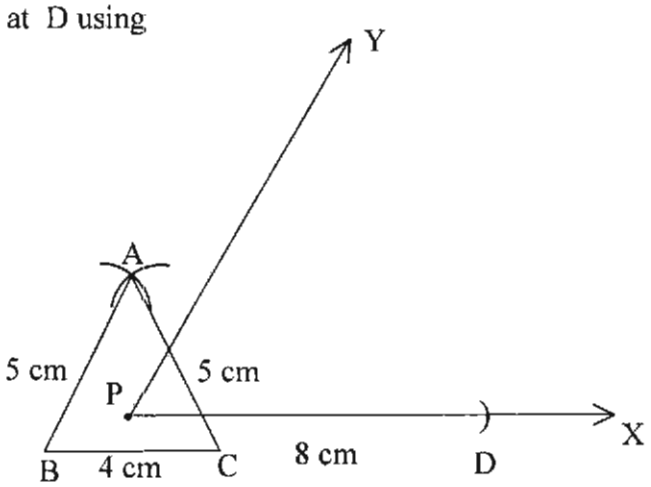


Fig. 6.63

Step 6: With P as centre and radius 10 cm ($5 \times 2 = 10$) draw an arc cutting the ray PY at E using compass. (Fig. 6.64).

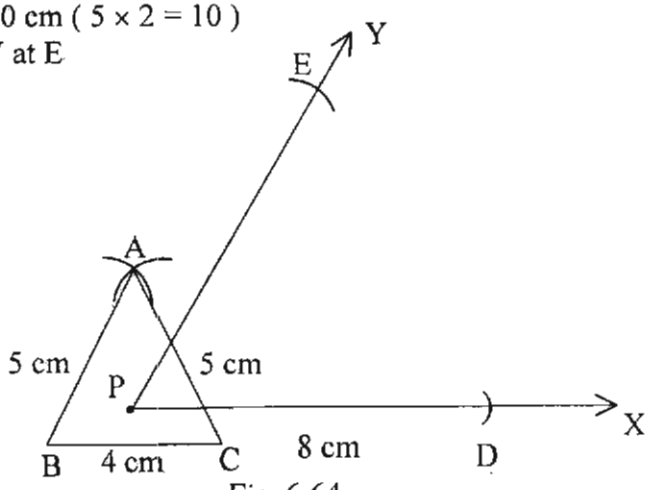


Fig. 6.64

Step 7: Join E and D. (Fig. 6.65) Now triangle EPD is the enlarged triangle of ABC in the ratio 1 : 2.

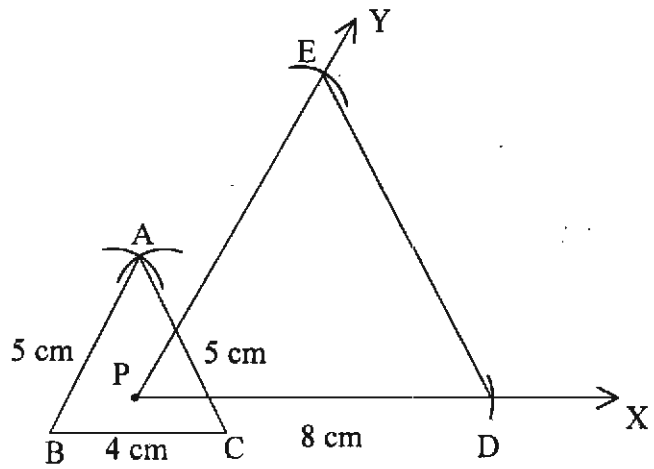


Fig. 6.65

Exercise 6.5

1. ***Construct an equilateral triangle ABC of side 5 cm and enlarge it in the ratio 1 : 2***
2. ***Construct a right triangle ABC in which $\underline{B}$ = 90°, BC= 2 cm , BA = 3 cm and enlarge it in the ratio 1 : 3***
3. ***Construct a triangle ABC in which AB = 4 cm , BC = 5 cm , AC = 6 cm and enlarge it in the ratio 1 : 2***
4. ***Construct an equilateral triangle ABC of side 6 cm . Enlarge the given triangle in the ratio 1 : 2 by taking any point inside the triangle as the centre of similitude.***
5. ***Construct an isosceles triangle ABC in which AB = AC = 6 cm and BC = 5 cm . Enlarge the given triangle in the ratio 1 : 2 by taking any point inside the triangle as the center of similitude.***
6. ***Construct a triangle ABC in which AB = 2 cm , BC = 3 cm and AC = 4 cm . Enlarge the given triangle in the ratio 1 : 3 by taking any point inside the triangle as the centre of similitude.***

7. HANDLING DATA

7.1 Data
7.2 Frequency distribution
7.3 Diagrammatic and graphic representation of a frequency distribution
7.4 Travel graphs

7.1 Data :

The marks obtained by the students of a class in a subject in an examination, the heights of the students of a class, the weights of the students of a class, the maximum temperatures in different places on a particular day in a state and the minimum temperatures in different places on a particular day in a state are collection of information in the form of numerical figures. **Any collection of information in the form of numerical figures giving the required information is called a data.**

Sometimes the numerical figures or the numerical facts are also called as scores, observations etc. We can also say that the word data means information in the form of numerical figures.

7.1.1 Raw data
7.1.2 Grouped data
7.1.3 Classification and tabulation of data

7.1.1 Raw data :

The marks obtained in a mathematics test by the students of a class is a collection of observations gathered initially. The information which is collected initially and presented randomly is called a raw data. Simply data obtained in the original form is called a raw data. **The raw data is an unprocessed and unclassified data.**

7.1.2 Grouped data :

Sometimes the collected raw data may be huge in number and it gives us no information as such. Whenever the data is very large, we have to group them meaningfully and then analyse. **The data which is arranged in groups or classes is called a grouped data.**

7.1.3 Classification and Tabulation of Data :

Grouping the collected data according to certain common properties is called classification of data. After classification of data the process of arranging numerical figures in groups or classes is called tabulation. Classification and tabulation of the data help us to get a better understanding of the data collected. Classification is the basis for tabulation. When the number of scores is large and the scores are repeated it is not convenient to write the scores individually. Hence the data collected is classified and tabulated.

7.2 Frequency distribution :

The number of times a particular score repeats itself is called its frequency. The tabular form of representing data showing each score and the corresponding frequency is called the frequency distribution.

7.2.1	**Frequency table**
7.2.2	**Frequency table for an ungrouped data**
7.2.3	**Frequency table for a grouped data**

7.2.1 Frequency table :

In order to tabulate a large number of scores, we use the grouped frequency table. In this method the scores are grouped or classified into small groups. These small groups are called class intervals. **The lower value of the class interval is called the lower limit and the upper value of the class interval is called the upper limit. The difference between the upper limit and the lower limit of a class interval is called the size or width of the class interval. The average of the upper limit and the lower limit is called the midvalue of that interval.**

In general, in a grouped frequency distribution, all class intervals are equal in size. The number of scores falling under each class interval is recorded by putting tally marks ('|') The number of tally marks in a given class interval represents the frequency of that class interval.

For example a frequency table is shown below.

Frequency Table

Class intervals	Tally marks	Frequency

Table 7.1

When we prepare a frequency table we have to follow some primary rules. It is desirable to have ten class intervals and as far as possible all the class intervals must be of equal size. After deciding about the choice of class intervals we should form a table with three headings namely class intervals, tally marks and frequency. We should read the raw data one by one and put the tally mark against the appropriate class interval. It should be clearly defined so that no doubt may be left as to which class interval a particular score belongs. When there are already four tally marks in a class interval like |||| and we have to enter the fifth tally mark in the same class interval it is entered as a cross - line cutting across diagonally all the four tally marks as shown 𝍸 and it is counted as five. We shall execute the above procedure in the following example. (See example 1)

7.2.2 Frequency table for an ungrouped data :

Example 1 :

Construct a frequency table for the following data .

5, 1, 3, 4, 2, 1, 3, 5, 4, 2,
1, 5, 1, 3, 2, 1, 5, 3, 3, 2.

Solution :

From the data, we observe that the numbers 1 , 2 , 3 , 4 and 5 are repeated. Hence under the number column write the five numbers namely 1 , 2 , 3 , 4 and 5 one below the other.

Now read the numbers one by one and put the tally mark in the tally mark column against the number. For example, the first number is 5. So put a tally mark (' | ') against the number 5. The next number is 1. So put a tally mark (' | ') against the number 1. Continue the process till all the numbers are exhausted.

Add the tally marks against the numbers 1 , 2 , 3 , 4 and 5 and write the total in the corresponding frequency column. Now add all the numbers under the frequency column and write it against the total.

Frequency table

Number	Tally marks	Frequency
1	𝍸	5
2	||||	4
3	𝍸	5
4	||	2
5	||||	4
	Total	20

Table 7.2

7.2.3 Frequency table for a grouped data :

Example 2 :

The following are the marks obtained by 50 students in a mathematics test. Prepare a frequency table for the data.

45	68	41	87	61	44	67	30	54	8
39	60	37	50	19	86	42	29	32	61
25	77	62	98	47	36	15	40	9	25
34	50	61	75	51	96	20	13	18	35
43	88	25	95	68	81	29	41	45	87

Solution :

To decide the length of the class interval and to take all the scores given in the problem, we have to find the largest value and the smallest value from the given scores. This we can do by merely going through all the scores. Here the largest value is 98 and the smallest value is 8.

$\therefore$ The difference = Largest value – Smallest value.

= 98 – 8

= 90.

Since the difference between the largest value and the smallest value is 90, taking class intervals such as 0 – 5 , 5 – 10 , 10 – 15 , ..., 95 – 100 , the class intervals are many in number (20). If we take the class intervals such as 0 – 10 , 10 – 20 , 20 – 30 ,....., 90 – 100, the class intervals are considerably reduced to 10. It is advisable not to overreduce the number of class intervals. There should be at least 5 class intervals. If the class intervals are mentioned in the problem then we proceed as given in the problem. For the above problem we form a frequency table taking class intervals 0 – 10 , 10 – 20 , 20 – 30 , 30 – 40 , 40 – 50 , 50 – 60 , 60 – 70 , 70 – 80 , 80 – 90 and 90 - 100.

The first score is 45. It lies in the class interval 40-50. Therefore put one tally mark (vertical bar like ' | ') in the class interval 40 – 50. The next score in the first row is 68 . It lies in the class interval 60 – 70 . Therefore put one tally mark in the class interval 60 – 70. In the same manner take the scores 41 , 87 , 61 , 44 , 67 one by one, identify the class intervals in which they lie and put a tally mark in the corresponding class interval.

Now the next score 30 is to be entered in a class interval. Here the doubt arises in which class interval 30 is to be included, either in the class interval 20 – 30 or 30 – 40 since the two class intervals contain the value 30. It is customary to include the value 30 in the class interval 30 – 40, which is the lower limit of the class interval. So put the tally mark for the score 30 in the class interval 30 - 40. As we continue the process we come across the score 61 in the second line, which is the 5th tally mark in the class interval 60 – 70. Mark the tally mark corresponding to 61 as 𝍸 in the class interval 60 – 70. If we have to put a 6^{th} tally mark in any class interval the tally mark is marked leaving a small gap after the block as shown 𝍸 |. This process is carried out till the last score is entered. The tally marks in each class interval are counted and the counted number is put against the same class interval under the frequency column. All the frequencies are added and the number is written as the total frequency for the entire class intervals. This must match the total number of data given. The table is called the frequency table for grouped data.

Frequency table of the marks taken by 50 students in a mathematics test.

Class Intervals	Tally Marks	Frequency
0 – 10	||	2
10 – 20	||||	4
20 – 30	𝍸 |	6
30 – 40	𝍸 ||	7
40 –50	𝍸 ||||	9
50 – 60	||||	4
60 – 70	𝍸 |||	8
70 – 80	||	2
80 – 90	𝍸	5
90 – 100	|||	3
	Total	50

Table 7.3

Exercise 7.1

1. ***Construct a frequency table for each of the following data***

   (a) *1 , 6 , 3 , 4 , 5 , 6 , 2 , 2 , 1 , 3 ,*
   5 , 3 , 6 , 4 , 4 , 1 , 3 , 1 , 2 , 1 .

   (b) *10 , 15 , 13 , 12 , 14 , 11 , 11 , 12 , 13 , 15 ,*
   11 , 13 , 12 , 15 , 13 , 12 , 14 , 14 , 15 , 11 .

2. *Fill in the blanks :*

(a) *Lower limit of the class interval 20 – 30 is* _____ .

(b) *Upper limit of the class interval 80 – 90 is* _____ .

(c) *Class – size of the class interval 50 – 60 is* _____ .

(d) *If the upper and lower limits of a class interval are 50 and 40 respectively then the class interval is* _____ .

(e) *If the lower and upper limits of a class interval are 60 and 70 respectively then the class interval is* _____ .

3. *The weights (in kg.) of 20 persons are given below :*

42 45 51 55 49 62 41 52 48 64

52 42 49 50 47 53 59 60 46 54

Form the grouped frequency table by taking class intervals 40 – 45 , 45 – 50 , 50 – 55 , 55 - 60, and 60 – 65 .

4. *The marks obtained by 30 students of a class in a mathematics test are given below:*

45 35 60 41 8 28 99 55 72 22

75 57 33 51 76 30 49 19 13 40

88 95 62 17 67 50 66 73 70 31

Form the grouped frequency table .

5. *On a particular day the earnings (in rupees) of 40 vendors are as under:*

37 41 39 36 73 28 59 60 59 80

83 72 66 40 75 42 38 58 33 55

39 50 43 72 80 44 79 67 52 39

72 63 50 63 59 61 40 77 68 75

Form the grouped frequency table.

6. *The marks obtained by 50 students of a class in a science test are given below:*

65 44 76 32 50 63 80 75 52 30

33 29 53 20 43 38 43 54 39 81

19 37 48 60 65 22 35 48 21 92

28 41 69 70 73 30 38 63 46 64

37 52 90 81 89 40 53 72 88 73

Form the grouped frequency table.

7.3 Diagrammatic and graphic representation of a frequency distribution :

7.3.1	**Histogram**
7.3.2	**Frequency polygon**
7.3.3	**Frequency polygon imposed on a histogram**

We have seen that classification and tabulation reduce the data to a simpler form. Even this simpler form of representation takes time for a common man to get the full meaning of the data. Hence for an easy understanding of the data in a tabular form, a visual aid in the form of graphs or diagrams becomes necessary. The following are some of the important graphic representations of a frequency distribution.

(a) Histogram

(b) Frequency Polygon

7.3.1 Histogram :

The histogram of a frequency data consists of a number of rectangles erected on the class intervals of the distribution. The areas of the rectangles are proportional to the frequencies of the respective classes. Usually the class intervals are marked successively along the X – axis and the frequencies are marked along the Y – axis. Our assumption is that the sum of the areas of all the rectangles of the histogram is the same as the total frequency.

Note : In all the graphic representations of this chapter , 2 subdivisions are taken as 1 cm.

7.3.1 (a) To draw a histogram for a given frequency table :

Example 3 :

Represent the following distribution by means of a histogram.

Marks	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	4	6	8	10	7	5

Table 7.4

Solution :

We know that when two mutually perpendicular axes are drawn on a graph paper four quadrants are formed. To draw a histogram we need only one quadrant namely the first quadrant since the class intervals and the frequencies are positive numbers. Hence take the X- axis along the bottom horizontal line and the Y-axis along the left extreme vertical line on the graph paper. Mark the class intervals continuously along the X-axis by taking a suitable scale. Identify the maximum frequency in the table and then choose a suitable scale along the Y-axis.

Write the scale on the right hand side corner of the graph sheet as shown in the fig. 7.1 Erect the first rectangle on the class interval 10 – 20 as the base with the height of 4 units (frequency). In the same manner erect the rectangles taking the consecutive class intervals as bases of the rectangles and their heights corresponding to the frequencies. The figure obtained is the required histogram (See fig. 7.1).

HISTOGRAM

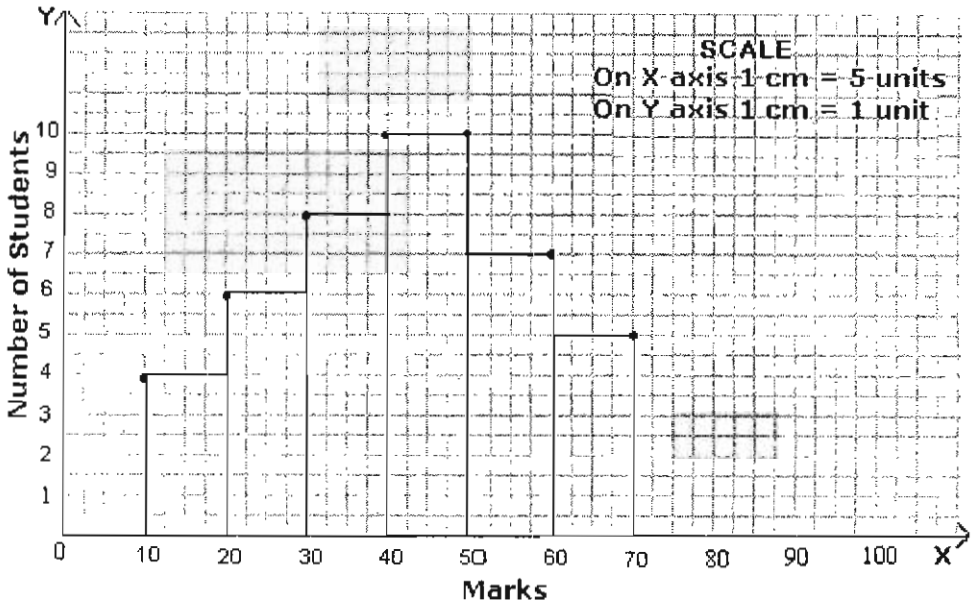


Fig. 7.1

7.3.2 Frequency Polygon :

Plot the points on a graph paper taking the midvalues of the class intervals as

x - coordinates and their corresponding frequencies as y - coordinates. Join the points with the help of line segments. We notice that the figure obtained is not a closed figure. Hence to get a closed figure mark the mid point of the class interval preceding the first class interval on the X-axis and join this point with the point corresponding to the midpoint of the first class interval. Mark the midpoint of the class interval succeeding the last class interval on the X-axis and join this point with the point corresponding to the midpoint of the last class interval. [See fig.7.2]. The closed figure thus obtained is called a frequency polygon.

7.3.2 (a) To construct a frequency polygon for a given frequency distribution :

Example 4 :

Construct a frequency polygon for the following distribution .

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 –70
Number of students	4	6	8	10	7	5

Table 7.5

Solution :

Mark the class intervals 0 – 10, 10 – 20, 20 – 30 ...60 – 70, 70 – 80 along the X-axis. The class intervals 0 – 10 and 70 – 80 are marked to get a closed frequency polygon as discussed above. Plot the points (5, 0), (15, 4), (25,6), (35,8), (45,10), (55,7), (65,5) and (75 , 0) on the graph paper. Join the points (5 , 0) and (15, 4) ; (15,4) and (25 , 6) ; (25 , 6) and (35, 8) ; (35,8) and (45,10);(45,10) and (55 , 7) ; (55 , 7) and (65 , 5) ; (65 , 5) and (75, 0) with the help of a ruler. Now we get a closed figure, which is the required frequency polygon. (See fig.7.2)

FREQUENCY POLYGON

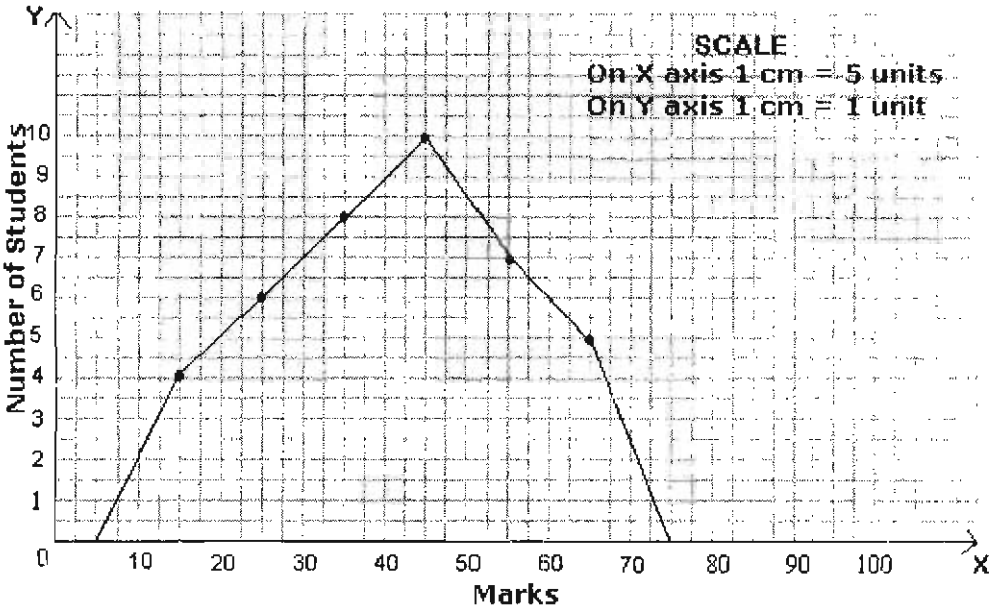


Fig. 7.2

7.3.3 Frequency polygon imposed on a histogram :

We now see how we can obtain a frequency polygon imposed on a given histogram. Construct the histogram for the given frequency table. Mark the mid points of the upper sides of the consecutive rectangles. Mark the midpoint of the class interval preceding the first class interval on the X-axis and also the midpoint of the class interval succeeding the last class interval on the X-axis. The consecutive marked points are joined with the help of a ruler. We get the frequency polygon imposed on the histogram. Now the sum of the areas of all the rectangles of the histogram is the same as the area enclosed between the X-axis and the frequency polygon.

7.3.3 (a) To construct a frequency polygon imposed on a given histogram :

Example 5 :

Construct a frequency polygon imposed on the histogram for the following distribution.

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of students	4	6	8	10	7	5

Table 7.6

Solution

Construct a histogram as we did in example 3. Now we mark the midpoints of the upper sides of the consecutive rectangles. We also mark the midpoint of the class interval 0 – 10 which is preceding the first class interval 10 – 20 and the midpoint of the class interval 70 – 80 which is succeeding the last class interval 60 – 70 on the X-axis. The consecutive marked points are joined with the help of a ruler. We get the frequency polygon imposed on the histogram (See fig.7.3)

FREQUENCY POLYGON IMPOSED ON A HISTOGRAM :

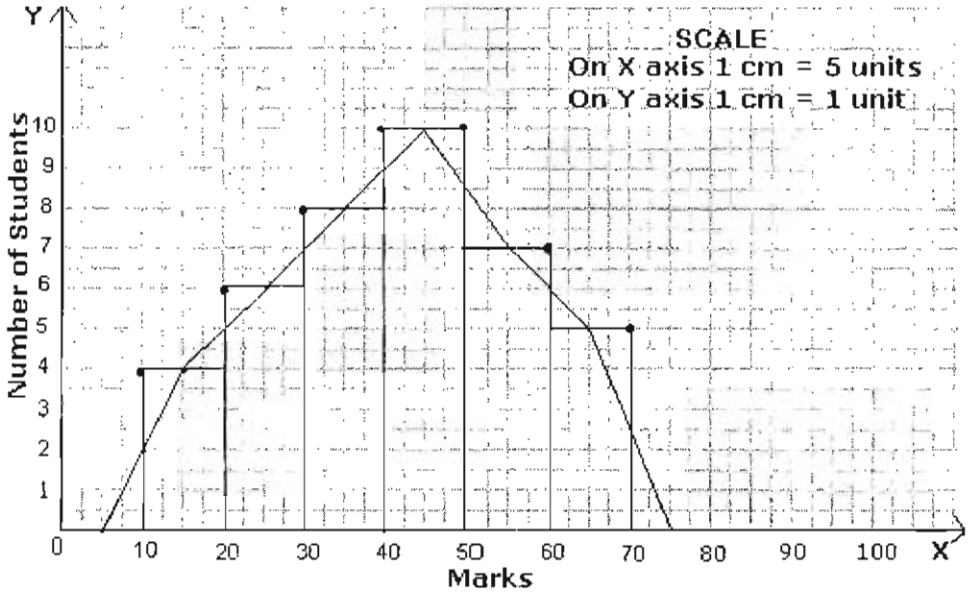


Fig.7.3

Exercise 7.2

1. *Draw a histogram to represent the following data*

Class intervals	*0 – 10*	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*
Frequency	*7*	*10*	*5*	*12*	*6*

2. *Draw a histogram to represent the following data.*

Age (in years)	*25 – 30*	*30 – 35*	*35 – 40*	*40 – 45*	*45 – 50*	*50 – 55*
Number of persons	*12*	*16*	*26*	*20*	*18*	*8*

3. *Construct the frequency polygon for the following frequency distribution.*

Class intervals	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*
Frequency	*8*	*16*	*22*	*18*	*6*

4. *Construct the frequency polygon for the following frequency distribution.*

Daily wages (in rupees)	*40 – 60*	*60 – 80*	*80 – 100*	*100 – 120*	*120 – 140*	*140 – 160*
Number of workers	*10*	*24*	*26*	*18*	*12*	*10*

5. *Construct a histogram and frequency polygon for the following data.*

Class intervals	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*	*60 – 70*
Frequency	*12*	*15*	*25*	*15*	*14*	*9*

6. The following are the marks obtained by 40 students in a science test. Prepare a frequency table for the data. Also draw the histogram and frequency polygon for the data.

15	*69*	*25*	*32*	*50*	*43*	*59*	*25*	*37*	*41*
23	*19*	*33*	*42*	*61*	*25*	*51*	*35*	*49*	*36*
41	*54*	*65*	*19*	*29*	*39*	*43*	*49*	*63*	*17*
35	*48*	*53*	*24*	*68*	*40*	*39*	*57*	*28*	*60*

THINGS TO REMEMBER

1. **Any collection of information in the form of numerical figures giving a required information is called a data.**
2. **Unprocessed and unclassified data is called a raw data.**
3. **The data which is arranged in classes or groups is called a grouped data.**
4. **The number of times a particular score repeats itself is called its frequency.**
5. **The tabular form showing each score and the corresponding frequency is called the frequency table.**
6. **The lower value of the class-interval is called the lower limit and the upper value of the class interval is called the upper limit of the class interval.**
7. **The difference between the upper limit and the lower limit of a class-interval is called the size or width of the class-interval.**
8. **The average of the lower limit and the upper limit is called the midvalue of that class-interval.**
9. **Histogram and frequency polygon are the two types of graphic representations of a frequency distribution.**
10. **In the graphic representation of histogram and frequency polygon the class-intervals are taken along the X-axis and the corresponding frequencies are taken along the Y-axis.**
11. **In a histogram the rectangles are placed continuously side by side with no gap between the adjacent rectangles.**
12. **The polygon obtained by joining the mid points of the top of the adjacent rectangles of a histogram and also the midpoints of the preceding and succeeding class intervals is known as a frequency polygon.**

7.4 Travel graphs :

Suppose there are forty students in a class. We give them each an identical rectangular shaped white paper and ask them to mark a point on the paper without consulting others. If we collect and analyse all the papers after the marking is over we can easily see that the positions of all the forty points will be different from one another. Why is this variation?

The obvious reason for this variation is that the students are not given any common guideline and instruction to mark the point and so the students mark the point at random. Now we understand that marking a point in a plane is very important and we need more informations to mark a point. (i.e) We need common guidelines.

7.4.1 Graph paper
7.4.2 To plot a given point on a graph paper
7.4.3 Line joining two given points
7.4.4 Linear graphs

7.4.1 Graph paper :

We have learnt the concept of number line in the chapter ' Number Work'. In fig. 7.4 $X'OX$ is a number line.

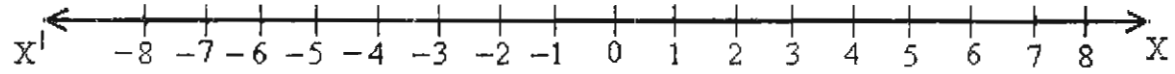


Fig. 7.4

All the numbers to the right of 0 (zero) are positive and all the numbers to the left of 0 (zero) are negative. We take another number line namely $Y'OY$ as shown in fig. 7.5

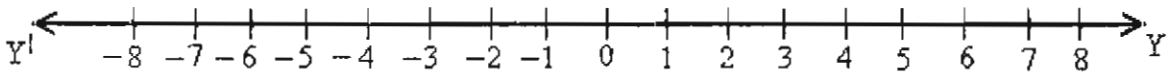


Fig. 7.5

We have also studied about the intersecting lines in theoretical geometry. We know that two lines intersect at only one point. Let the two number lines $X'OX$ and $Y'OY$ intersect at right angles at 0 (zero) as shown in fig. 7.6

We call the number line $X'OX$ as the X-axis and the number line $Y'OY$ as the Y- axis. We know how to read the numbers on the number line. Suppose we have to plot a point, which is at a distance of 5 units from the Y-axis and 3 units from the X-axis.

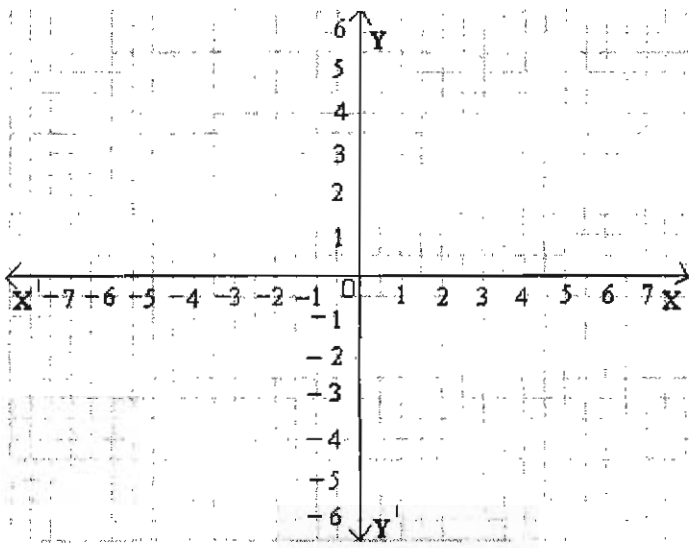


Fig. 7.6

Start from 0 (zero or origin) and go along the X-axis up to 5 units. From there go upward 3 units parallel to the Y axis and mark this position as point P. The position of P is (5 , 3) and it is denoted as P (5 , 3). The distance travelled along the X-axis is called abcissa or X coordinate of the point (5 , 3). The distance travelled parallel to the Y axis is called the ordinate or Y coordinate of the point (5 , 3). Any point is marked by a pair of numbers written within a bracket as shown above. If these guidelines and instructions are given to the students, all the students mark the point (5 , 3) in the same position.

The two mutually perpendicular axes $X'OX$ and $Y'OY$ divide the plane into four quadrants as in fig. 7.7

The first quadrant is the plane, which lies to the right of the Y-axis and above the X-axis. The other three quadrants are numbered as II, III and IV in the anticlockwise direction starting from the I quadrant. In the first quadrant both X and Y coordinates are positive. In the second quadrant the X coordinate is negative and the Y coordinate is positive. In the third quadrant both X and Y coordinates are negative. In the fourth quadrant the X coordinate is positive and the Y coordinate is negative.

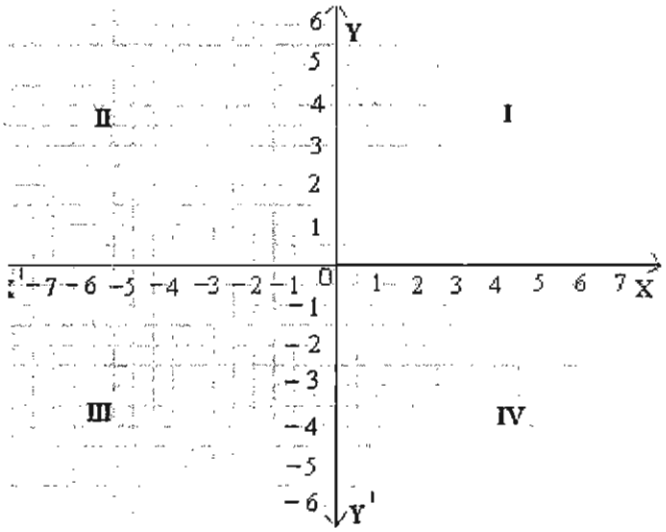


Fig. 7. 7

If the X coordinate and the Y coordinate of a point are given by means of a pair of numbers within a bracket the point is unique. In a graph paper the plane is uniformly divided into small uniform squares with side 1 cm , the sides of each square touching the sides of the neighbouring squares. Further each square is divided into 100 small squares of side 0.1 cm or 1 mm. The parallel sides of the squares are shown by thick lines.

7.4.2 To plot a given point on a graph paper :

Example 6 :

Plot the following points on a graph paper.

(a) (3 , 4) (b) (– 2 , 3) (c) (– 3 , – 4) (d) (4 , – 3)

Solution :

(a) Point P (3 , 4)

Here the X-coordinate of P is 3 and the Y-coordinate of P is 4 and both are positive. Hence the point P (3 , 4) lies in the I quadrant. We first take 3 units to the right along the X- axis and then move upward 4 units at right angles to OX and mark the point P (3 , 4) as shown in fig. 7.8.

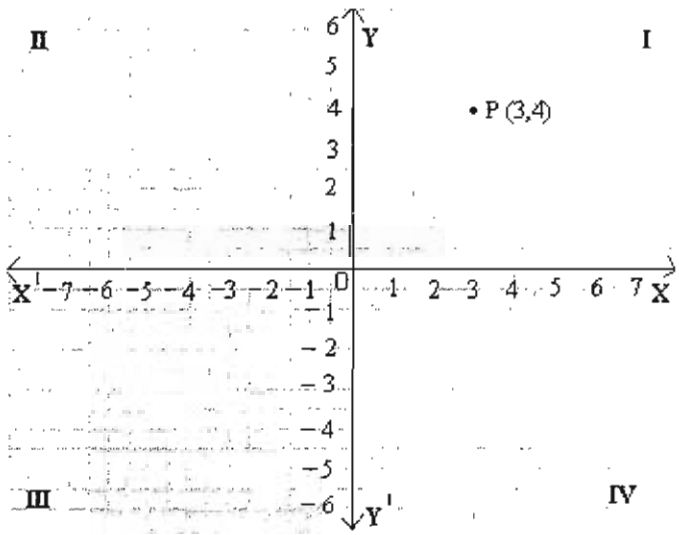


Fig. 7.8

(b) Point Q (– 2 , 3)

Here the X- coordinate of Q is –2 which is negative and the Y-coordinate of Q is 3 which is positive. Hence the point Q (– 2 , 3) is in the II quadrant. We first take 2 units to the left along OX' because it is negative and then move upwards 3 units at right angles to OX' and mark the point Q (– 2 , 3) as shown in fig. 7.9

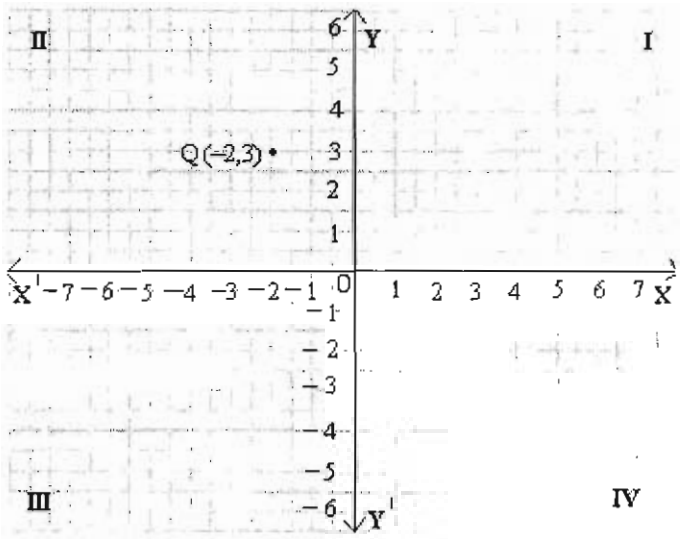


Fig. 7.9

(c) Point R (– 3 ,– 4)

Here the X- coordinate of R is - 3 which is negative, and the Y-coordinate of R is – 4 which is also negative. Hence the point R (– 3 , – 4) is in the III quadrant. We first take 3 units to the left along OX' because it is negative and then move downwards 4 units at right angles to OX' and mark the point R (– 3 , – 4) as shown in fig. 7.10

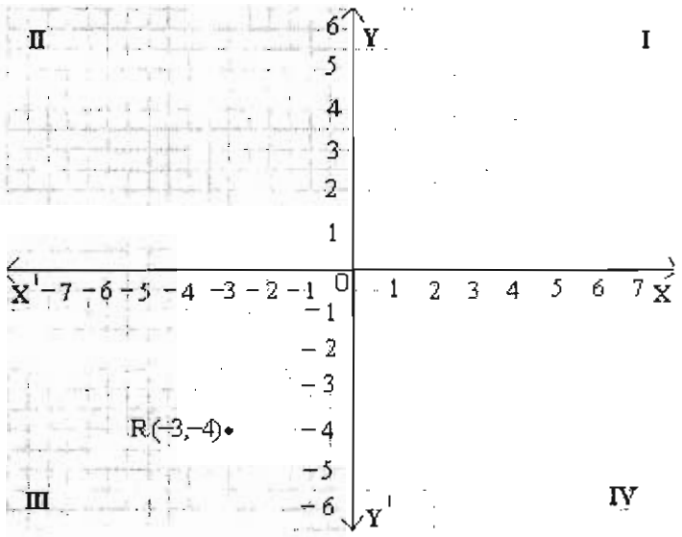


Fig. 7.10

(d) Point S (4 , – 3)

Here the X- coordinate of S is 4 which is positive and the Y-coordinate of S is – 3 which is negative. Hence the point S (4, –3) lies is in the IV quadrant. We first take 4 units to the right along OX and then move downward 3 units at right angles to OX and mark the point S (4, – 3) as shown in fig. 7.11

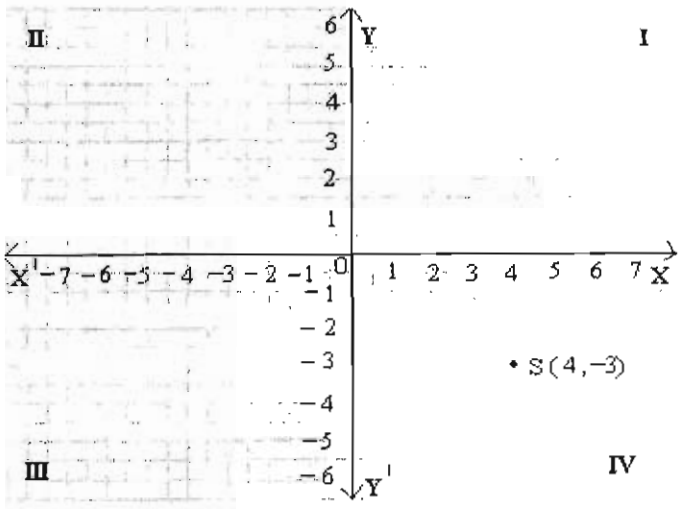


Fig. 7.11

7.4.3 Line joining two given points :

Example 7 :

Draw the line joining the following points.

(a) (2 , 3) and (4 , 5) (b) (– 3 , – 3) and (3 , 6) (c) (4 , 2) and (4 , – 6)

(d) (– 2 , 5) and (6 , 5)

Solution :

(a) Draw the line joining the points (2 ,3) and (4 , 5).

Plot the point (2 , 3) and denote it as A. Plot the point (4 , 5) and denote it as B

Join the points A and B

∴ AB is the required line (See fig. 7.12)

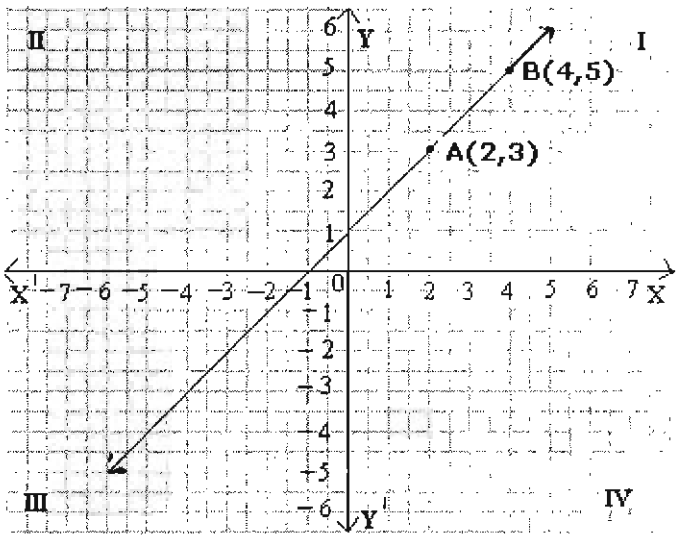


Fig. 7. 12

(b) Draw the line joining the points (– 3 , – 3) and (3 , 6) .

Plot the point (– 3 , – 3) and denote it as C. Plot the point (3 , 6) and denote it as D. Join the points C and D.

∴ CD is the required line.

(See fig. 7.13)

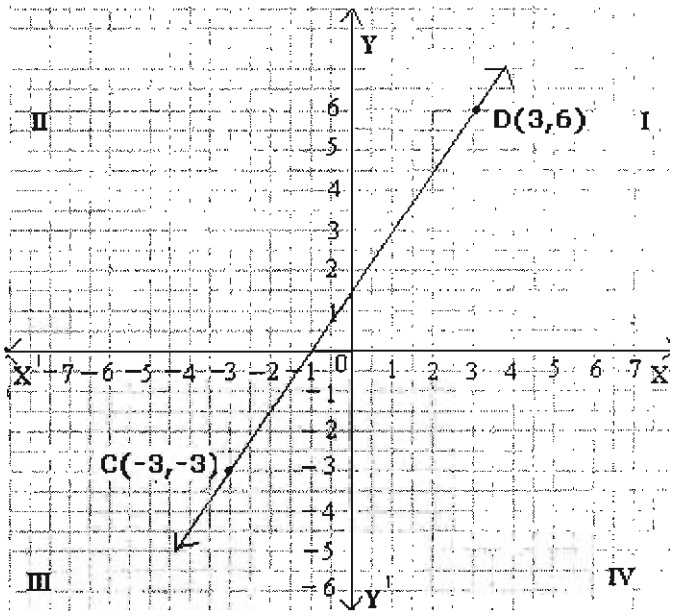


Fig. 7.13

(c) Draw the line joining the points (4 , 2) and (4 , – 6) .

Plot the point (4 , 2) and denote it as E. Plot the point (4 , – 6) and denote it as F

Join the points E and F.

∴ EF is the required line.

(See fig. 7.14)

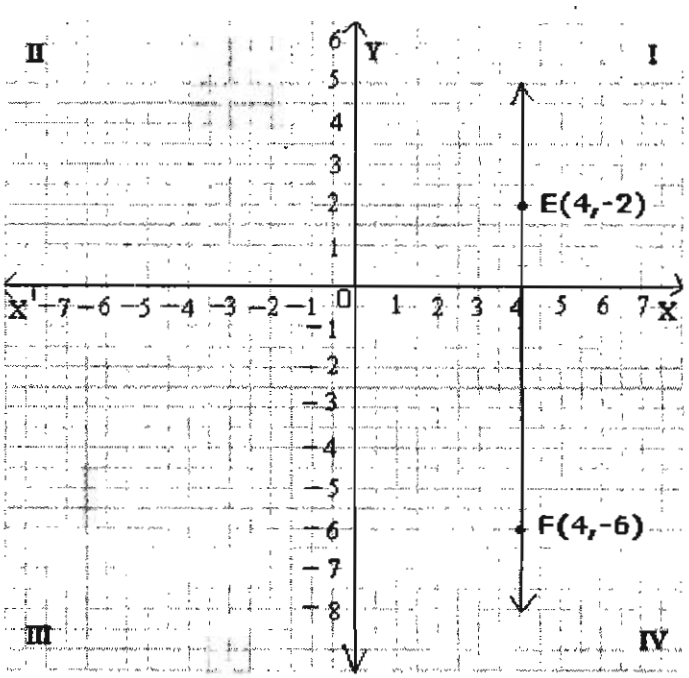


Fig. 7.14

(d) Draw the line joining the points (– 2 , 5) and (6 , 5).

Plot the point (– 2 , 5) and denote it as G. Plot the point (6 , 5) and denote it as H.

Join the points G and H.

∴ GH is the required line.

(See fig. 7.15)

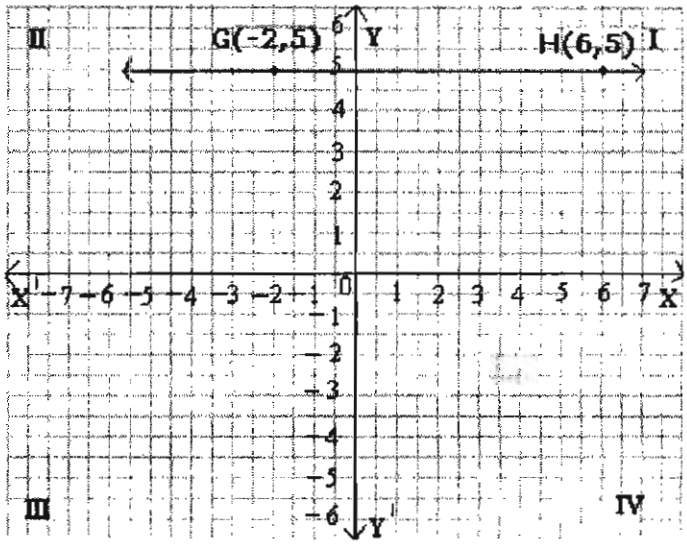


Fig. 7.15

7.4.4 Linear graphs :

A graph is a kind of picture, which shows clearly the relation between two quantities. A graph shows how one quantity changes as the other quantity changes. The relation between time and distance can be shown by a graph. This graph is known as travel graph. **If the relation between any two quantities can be represented by means of a straight line in a graph paper then such graphs are known as linear graphs.**

7.4.4 (a) Time – Distance graph :

Time - distance graph is a linear graph. Suppose a man travels at a uniform speed of 40 km per hour. In one hour he will be at a distance of 40 km from the starting point. In two hours he will be at a distance of 80 km from the starting point. In three hours he will be at a distance of 120 km from the starting point and so on. In the above example if the time

increases the distance travelled also increases and if the time decreases the distance travelled also decreases. Here time and distance are considered as two variables which are in direct variation. Therefore for direct variation the increase or the decrease between the variables must be in a constant proportion. The relation between time and distance can be shown in a graph by taking time along the X- axis and distance along the Y-axis. The graph obtained is a straight line. This graph is known as 'Time – Distance graph'

7.4.4 (b) A linear graph showing the relationship between time and distance :

Example 8 :

Draw the linear graph for the following table

Time (in hours)	1	2	3	4	5	6
Distance (in km.)	20	40	60	80	100	120

Table 7.7

Solution :

Here both the quantities time (in hours) and distance (in km.) are positive. Taking the time along the X- axis and the distance along the Y-axis , the table can be converted as points (1 , 20) , (2 , 40) , (3 , 60) , (4 , 80) , (5 , 100) , (6 , 120). In the above points all the X- coordinates and the Y-coordinates are positive. Therefore to draw the graph the I quadrant is enough. Hence we take only the I quadrant on the graph paper.

Time –distance graph

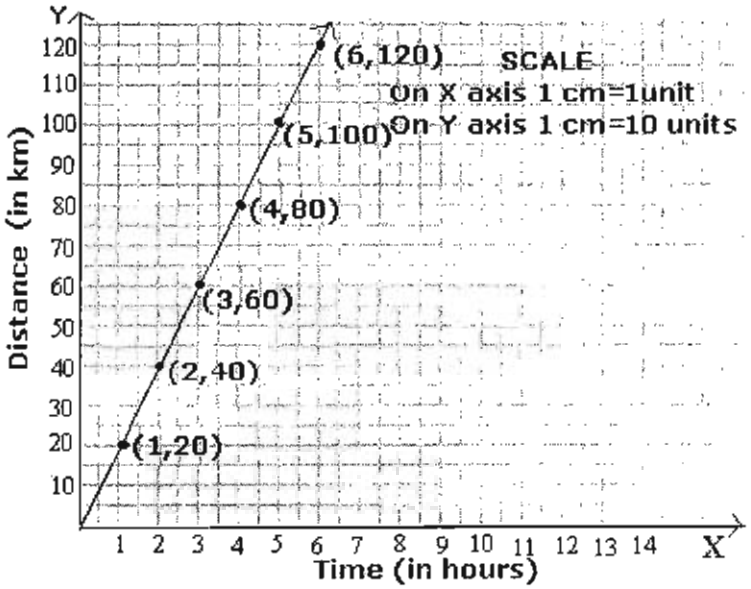


Fig. 7. 16

Draw the X-axis and the Y-axis . Take a suitable scale. Plot the points (1,20) , (2,40), (3,60) , (4, 80), (5, 100) and (6, 120). Join the points and we get a straight line. This graph shows the relation between time and distance. (See fig. 7.16)

Example 9:

A man travels at a uniform speed of 25 km per hour. To show the time-distance relation draw a linear graph.

Time (in hours)	1	2	3	4	5	6
Distance (in km)	25	50	75	100	125	150

Table 7.8

Solution :

Here both the quantities time and distance are positive. Taking the time along the X- axis and the distance along the Y-axis , the table can be converted as points (1,25), (2,50) , (3 , 75) , (4 , 100) , (5 , 125) , (6 , 150). In the above points all the X- coordinates and the Y-coordinates are positive. Therefore to draw the graph the I quadrant is enough. Hence we take only the I quadrant on the graph paper. Draw the X – axis and the Y- axis. Take a suitable scale. Mark the points (1,25), (2,50), (3,75), (4,100), (5,125) and (6,150). Join the points. We get a straight line. This graph shows the relation between time and distance.

(See Fig. 7.17)

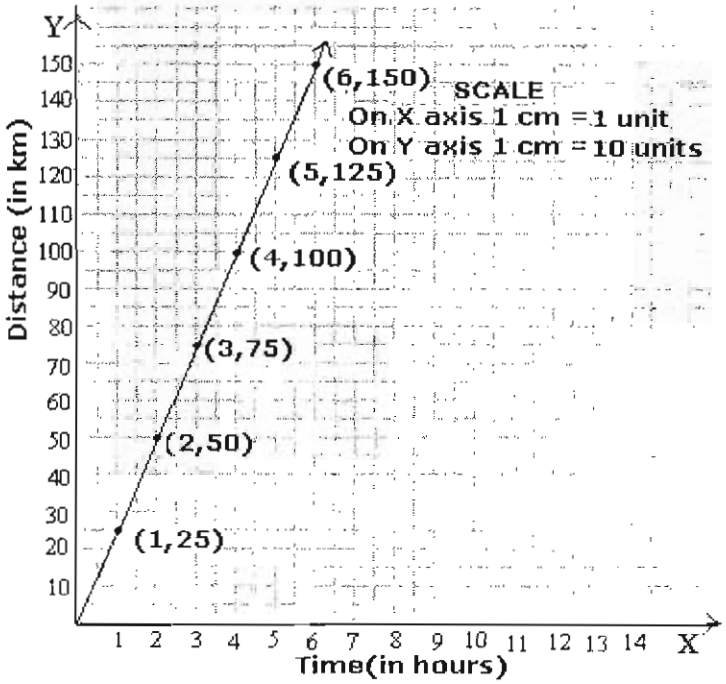


Fig. 7. 17

Example 10 :

A train travels at a uniform speed of 50 km per hour. To show the time-distance relation draw a linear graph.

Solution :

From the problem we understand that every one hour the distance increases by 50 kms. Therefore the relation between time and distance can be shown in the form of a table given below:

Time (in hours)	1	2	3	4	5	6	7	8
Distance (in km.)	50	100	150	200	250	300	350	400

Table 7.9

Draw the X-axis and the Y-axis. Take a suitable scale. Plot the points (1,50),(2,100),(3, 150) , (4, 200) , (5, 250) , (6, 300), (7, 350) and (8, 400). Join the points and we get a straight line. This graph shows the relation between time and distance.

[see fig. 7.18]

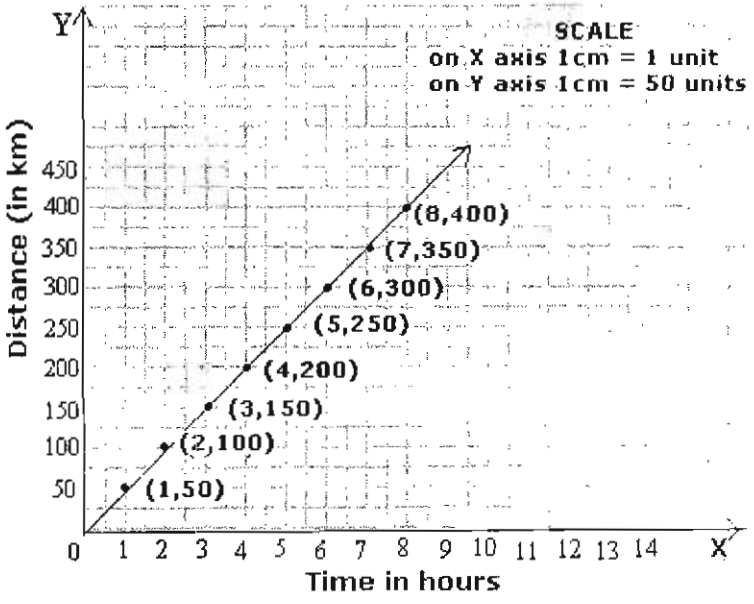


Fig. 7.18

Exercise 7.3

1. Plot the following points on a graph paper.

(a) (6 , 8) *(b) (– 8 , 6)* *(c) (– 6 , – 7)* *(d) (5 , 4)*

(e) (4 , 4) *(f) (– 3 , 3)*

2. Draw the line joining the following points.

(a) (3 , 4) and (– 3 , 5) *(b) (4 , 8) and (– 8 , – 8)*

(c) (– 4 , 6) and (5 , – 3) *(d) (2 , 7) and (–7 , 2)*

3. Draw a linear graph for the following table.

Time (in hours)	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Distance (in km.)	*30*	*60*	*90*	*120*	*150*	*180*	*210*	*240*

4. A man travels at a uniform speed of 40 km per hour. To show the time – distance relation draw a linear graph.

5. A woman travels at a uniform speed of 35 km per hour. To show the time – distance relation draw a linear graph

THINGS TO REMEMBER

1. **The X- axis and the Y – axis are two graduated lines intersecting at right angles at 0 (origin or zero).**
2. **On the X- axis the measurements increase from left to right and on the Y-axis the measurements increase from bottom to top.**
3. **The X-axis and the Y-axis divide the plane into four quadrants.**
4. **The quadrant which is to the right of the Y-axis and above the X-axis is the I quadrant.**
5. **The other quadrants namely II , III, and IV are marked in the anticlockwise direction from the I quadrant.**
6. **A point P on a graph paper is marked as P (x , y) where x is the X- coordinate and y is the Y – coordinate.**
7. **The position P in the I, II, III and IV quadrants are respectively P (x , y) , P (– x , y) , P (– x , – y) and P (x , – y).**
8. **A graph is a kind of diagram which shows clearly the relation between two quantities.**
9. **The graph showing the relation between time and distance is known as travel graph.**
10. **The straight line in a graph paper showing the relation between two quantities is known as linear graph.**

ANSWERS

Exercise 1.1

1. (a) 1 ,2 , 3 ,4 (b) 0 ,1 , 2 , 3 (c) – 3, – 2 , –1 (d) – 5 , – 4, – 3
2. (a) – 5 (b) 18 (c) – 33 (d) 0
3. (a) – 5 , – 3 , – 1 , 0 , 2 , 4 (b) – 7 , – 5 , – 3 , 4 , 5 , 6
4. (a) 4 , 2 , 1 , 0 , – 3 , – 7 (b) 8 , 6 , 5 , 0 , – 6 , – 10

Exercise 1.2

1. (a) – 152 (b) – 590 (c) –10 (d) 18 (e) 14 (f) – 30
2. (a) – 60 (b) – 5 (c) 57 (d) 471
3. (a) – 140 (b) 180 (c) – 800 (d) – 1342
4. (a) – 8 (b) - 32 (c) 9 (d) – 21
5. (a) – 30 (b) – 12 (c) 300
6. (a) + 5 (b) 0 (c) + 15 (d) – 5 (e) – 12

Exercise 1.3

(1) 20 (2) – 5 (3) – 4 (4) 10 (5) 47
(6) 800 (7) 96 (8) 60 (9) – 19 (10) 1

Exercise 1.4

1. (a) $\frac{4}{7}$ (b) $\frac{8}{15}$
2. (a) $\frac{3}{5}$ (b) $\frac{1}{3}$ (c) $\frac{5}{3}$ (d) $\frac{1}{9}$
3. (a) $\frac{17}{18}$ (b) $\frac{27}{20}$ (c) $6\frac{2}{9}$ (d) $\frac{83}{11}$
4. (a) $-\frac{1}{26}$ (b) $\frac{5}{24}$
5. (a) $\frac{4}{5}$ (b) $2\frac{11}{12}$
6. (a) $\frac{19}{24}$ (b) $1\frac{1}{3}$ (c) $\frac{5}{12}$ (d) $1\frac{1}{120}$
7. (a) $\frac{3}{25}$ (b) 6 (c) $\frac{1}{10}$ (d) 33
8. (a) $\frac{1}{4}$ (b) 30 (c) $\frac{3}{19}$ (d) 35

Exercise 1.5

1. (a) $-\frac{8}{14}$ (b) $-\frac{12}{21}$

2. (a) $-\frac{10}{15}$ (b) $\frac{3}{13}$ (c) $-\frac{9}{4}$ (d) $\frac{1}{4}$

3. (a) $-\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $-\frac{3}{5}$ (d) $-\frac{5}{3}$

4. (a) $\frac{21}{19}$ (b) $-\frac{13}{6}$ (c) $-\frac{17}{5}$ (d) $-\frac{109}{7}$

5. (a) $3\frac{1}{8}$ (b) $-3\frac{4}{5}$ (c) $-2\frac{5}{12}$ (d) $+3\frac{5}{6}$

6. (a) $\frac{-2}{3} = \frac{-6}{9} = \frac{10}{-15}$ (b) $\frac{-3}{-8} = \frac{+6}{16} = \frac{9}{24}$

(c) $\frac{-4}{13} = \frac{-12}{39} = \frac{20}{-65}$ (d) $\frac{-6}{-10} = \frac{3}{5} = \frac{-12}{-20}$

7. (a) $\frac{-15}{20} > \frac{25}{-30}$ (b) $\frac{-7}{21} = \frac{3}{-9}$ (c) $\frac{-8}{-14} < \frac{13}{21}$ (d) $\frac{-6}{10} > \frac{11}{-15}$

8. (a) $\frac{-4}{7}$ (b) $\frac{-5}{8}$ (c) $\frac{-7}{12}$ (d) $\frac{4}{-3}$

Exercise 1.6

1. (a) -1 (b) $\frac{-4}{3}$ (c) $\frac{-10}{17}$ (d) -1

2. (a) $\frac{3}{28}$ (b) $\frac{-32}{39}$ (c) $\frac{1}{78}$ (d) $\frac{-4}{15}$

3. (a) $\frac{8}{11}$ (b) $\frac{-3}{4}$ (c) $\frac{-57}{40}$ (d) $\frac{-523}{132}$

4. (a) $\frac{-3}{8}$ (b) $\frac{-5}{2}$ (c) -15 (d) $\frac{2}{9}$

5. (a) -5 (b) $\frac{-1}{4}$ (c) $\frac{33}{5}$ (d) $\frac{-18}{5}$

6. (a) $\frac{1}{12}$ (b) $\frac{-3}{4}$ (c) $\frac{8}{7}$ (d) -30

7. $\frac{21}{20}$ 8. $\frac{77}{72}$ 9. $\frac{22}{13}$ 10. $\frac{-14}{15}$

Exercise 1.7

1. (a) 169 (b) 324 (c) 484 (d) 15129

2. (a) 17 (b) 19 (c) 80 (d) 56

3. (a) 12 (b) 19 (c) 100 (d) 1

4. (a) 48 (b) 24 (c) 49 (d) 16

5. (a) 64 (b) 216 (c) 1728 (d) 50653

6. (a) 5 (b) 7 (c) 32 (d) 25

7. (a) 7 (b) 9 (c) 10 (d) 16 (e) 14 (f) 18

Exercise 1.8

1. (a) 42 (b) 986 (c) 502 (d) 1059 (e) 547 (f) 200

2. (a) 8.4 (b) 12.8 (c) 24.0 (d) 5.2

3. (a) 41.40 (b) 512.03 (c) 16.56 (d) 4.35

4. (a) 2.154 (b) 57.885 (c) 27.300 (d) 30.000

5. (a) 300 and 340 (b) 3000 and 2700 (c) 10 and 13 (d) 5000 and 5500
(e) 2000 and 2000

6. (a) 2 (b) 2 (c) 1 (d) 3 (e) 1 7. (a) 4 (b) 1

Unit Test -1

Part I

1. (c) + 7 2. (d) - 4 3. (b) - 1 4. (a) 4 5. (b) 24.65

6. 10 7. $\frac{-2}{5}$ 8. $\frac{-4}{5}$ 9. 9 10. 3

Part II

11. $-4, -3, -2$ 12. -37 13. 1 14. $\frac{7}{8}$ 15. $\frac{-5}{17}$

16. $\frac{-4}{8} = \frac{+8}{-16} = \frac{-12}{24}$ 17. $\frac{1}{96}$ 18. -2

19. 8 20. 700 and 740

Part III

21. -108 22. $\frac{-7}{13}$ 23. -17 24. -5 25. 25

Exercise 2.1

1. 25 2. 9 3. 10 4. 8 5. 2
6. 25 7. 8 8. 40 9. 40 10. 6

Exercise 2.2

1. (a) 75 % (b) 50 % (c) 40 % (d) 320 % (e) 250 %
(f) 425 % (g) 80 % (h) 25 % (i) 150%

2. (a) $\frac{3}{5}$ (b) $\frac{7}{20}$ (c) $\frac{4}{5}$

3. (a) 40 (b) 40 (c) 3.75

4. (a) 90 % 5. Rs. 5000

Exercise 2.3

1. Rs. 1000 2. Rs. 2500 3. Rs. 8000 4. 4 %

5. $2\frac{1}{2}\%$ 6. 2% 7. Rs. 1,50,000 8. Rs. 1,00,000 9. Rs. 2,27,250

10. (a) Rs. 160 (b) 3 % (c) Rs. 10,000

Exercise 2.4

1. Rs. 855 2. Rs. 127.50 3. Rs. 13,775 4. 10 %

5. $2\frac{1}{2}\%$ 6. $16\frac{2}{3}\%$ 7. $16\frac{2}{3}\%$ 8. Rs. 150 9. Rs. 1,000

10. (a) Rs.12 (b) Rs.450 (c) Rs.12,000

Exercise 2.5

1. Rs. 5,600 2. Rs. 24,000 3. Rs. 17,025 4. Rs. 29,940

5. 30 % 6. 10 %

Exercise 2.6

1. Rs. 4,500 2. Rs. 23,500 3. Rs. 13,150 4. 6 % 5. 8 %

6. 5 % 7. 2 8. 4 9. 2 10. Rs. 1,000

11. Rs. 5,000 12. 10 %

13. (a) Rs. 250 , Rs. 2750 (b) 3 , Rs.4,720 (c) 4 % , Rs. 8,250

(d) Rs. 5,000, Rs. 7,000 (e) Rs. 12,000 , $1\frac{1}{2}$

Exercise 2.7

1. Rs. 28,170 2. Rs. 10,800 3. Rs. 4,575 4. 8 % 5. 6 %

6. 9 %

Unit Test - 2

Part - I

1. 4 2. Rs. 100 3. Rs. 70 4. Rs. 600 5. Rs. 500

6. Inverse 7. Discount 8. Nil 9. Rs. 300 10. p n + I

Part - II

11. 4 12. Rs. 1000 13. $8\frac{1}{3}\%$ 14. 4 15. Rs. 915

Part - III

16. Rs. 150 17. Rs. 2000 18. Rs. 16,700 19. Rs. 13,300 20. 6 %

Exercise 3.1

1. (a) 09 . 30 hrs (b) 12. 00 hrs (c) 16.10 hrs (d) 24.00 hrs or 00.00 hrs
2. (a) 8.15 p.m. (b) 8.40 a.m. (c) 12.30 p.m. (d) 10.35 a.m
3. 3 hrs 5 mins.
4. (a) Mayiladuthurai – Karaikudi – Mayiladuthurai
   (b) Down train numbers : 6713 and 861 ; Up train numbers : 6714 and 862
   (c) 121 km. (d) 20.40 hrs or 8.40 p.m ; 5 mins.
5. 5 hrs 5 mins. and 6 hrs 10 mins. ; Train No. 862

Exercise 3.2

1. (a) 137 ; Chennai – Madurai (b) Tindivanam, Villupuarm, Thozhudur and Trichy
   (c) 197 km. (d) 15 mins. (e) 9 hrs 55 mins.
2. (a) 7 hrs (b) Chennai – Madurai (c) 8.30 p.m. or 20:30 hrs
   (d) 6 hrs (c) Trichy and Kanyakumari
3. Train journey is faster. No, the faster journey is not the cheaper journey

Exercise 3.3

1. (a) $l\,b$ (2) $2\,(l+b)$ (c) a^2 (d) $4a$ (e) $\frac{1}{2}\,bh$

2. (a) A = 120 sq. m ; P = 46 m
   (b) = 1.92 sq. m (or 19200 sq. cm) ; P = 5.6 m (or 560 cm.)
3. (a) A = 576 sq. cm ; P = 96 cm (b) A = 2.25 sq. m ; P = 6 m
4. (a) A = 60 sq. cm (or 6000 sq. mm) (b) A = 1.6 sq. m
5. 60 ares 6. 200

Exercise 3.4

1. 120 cm^2
2. (a) 60 m^2 (b) 6 sq. m (or 600 sq. dm) c) 10.5 sq. cm (or 1050 sq. mm)
3. (a) 60 cm, (or 6 dm) (b) 15 m 4. (a) 7 cm (b) 10 dm
5. 1.6 hectares

Exercise 3.5

1. (a) 2200 cm^2 (b) 105 dm^2 (or 10500 cm^2) (c) 640 dm^2
2. 36 m 3. 10 cm 4. Rs. 3000 5. 3 hectares

Exercise 3.6

1. 480 cm^2 2. (a) 160 cm^2 (b) 84 cm^2 (c) 30.6 m^2 3. 5 m
4. Rs. 4800 5. 40 m

Exercise 3.7

1. (a) 63 cm^2 (b) 15.2 dm^2 (or 1520 cm^2) (c) 60 cm^2
2. 12 m 3. 6. 5 cm 4. 1500 dm^2 5. 5. 4 ares

Exercise 3.8

1. (a) 88 cm (b) 176 cm (c) 30.8 m
2. (a) 110 cm (b) 132 dm (c) 19.8 m
3. (a) 70 mm (b) 21 cm (c) 8.4 m
4. (a) 182 cm (b) 24.5 m (c) 1 m
5. 1100 cm 6. Rs 2200 7. 14 cm 8. 84 m
9. 3300 cm 10. 140 cm

Exercise 3.9

1. (a) 1386 cm^2 (b) 38.5 cm^2 (c) 616 sq. dm
2. (a) 154 m^2 (b) 86.625 cm^2 (c) 962.5 sq. dm
3. 13.86 cm^2 4. 94.5 cm^2 5. 38.5 m^2 6. 154 m^2
7. 3.5 m 8. Rs. 6160 9. Circle , 132 cm^2 10. 108.5 cm

Exercise 3.10

1. (a) 77 cm^2, 36 cm (b) 173.25 cm^2 , 54 cm (c) 19.25 cm^2 , 18 cm
2. (a) 1386 cm^2 , 150 cm (b) 154 cm^2 , 50 cm (c) 24.64 cm^2 , 20 cm
3. 38.5 cm^2 4. 12.32 cm^2 , 6.16 cm^2 5. 346.5 cm^2

Exercise 3.11

1. 336 m^2 2. Rs. 10,400 3. Rs. 12.80 4. 4.25 m^2 5. 0.56 m^2
6. (a) 7 cm (b) 3.5 m (c) 5 dm 7. 2310 m^2 8. Rs. 2425.50
9. 731.5 m^2 10. 16.14 m^2

Exercise 3.12

2. (a) 15 cm^3 (b) 210 cm^3 (c) 120 m^3 (d) 6325 cm^3 (e) 18 m^3
3. (a) 3 cm (b) 25 cm (c) 15 m (d) 20 cm
4. 300 cm^3 5. 5000 6. 22,500 lit.
7. (a) 512 cm^3 (b) 42.875 m^3 (c) 15625 mm^3

Unit Test - 3

Part I

1. (b) $l\,b$ sq. units 2. (a) 100 m^2 3. (c) 14 cm
4. (a) $(\pi + 2)$ r units 5. (d) $l\,b\,h$ cu. units 6. 6 7. 1.15 p.m.
8. $\frac{1}{2}$ $d_1\,d_2$ sq. units 9. 3 cm 10. $\frac{\pi r^2}{4}$ sq. units

Part II

11. 7 hrs 45 mins 12. 36 cm 13. 30 cm^2 14. 24 cm
15. 54 cm^2 16. 2 m 17. 80 cm^3 18. 216 m^3
19. (a) 7.40 p.m (b) 07. 25 hrs 20. (a) 125 are (b) 25000 m^2

Part III

21. Nellai express train route , No 22. 12 cm 23. Rs. 552
24. 3000

Exercise 4.1

1. (a) $5x = 15$ (b) $x^2 = 36$ (c) $10 + x = 22$
(any variable from a to z may be used)
2. Constants : 15 , – 8 , 7. 5 ; variables : a , x^2 , m^3 , p , – q

3. The base and power are

(a) a and 3 (b) m and 2 (c) q and 1 (d) y and 5 (e) x and 4

4. a) 5 (b) -7 (c) $\frac{-3}{8}$ (d) 1 (e) -27

5. Like terms : (a) 3 x and 5 x (c) $3a^2b$ and $-6a^2b$ (d) $-8y^3$ and $3y^3$

Unlike terms : (b) 7 x and -5 y (c) $10p^2q$ and $8pq^2$

6. (a) 12 x (b) x^2y (c) 13 m (d) 11 a b (e) $4x^2$

7. (a) 5 x y (b) $5a^2$ (c) $-9y$ (d) 3 a b c (e) $-2x^2y$

Exercise 4.2

1. (a) $x^2 - 5yz$ (b) $2 + m + n$ (d) $11a^2 + 6b^2 - 5$

2. (a) 4 (b) 3 (c) 1 (d) 2 (e) 3 (f) 2

3. (a) $x^3 - 8$ (b) $2a^2 - 5a - 7b$

(d) $4p^3 - 7p^2 + 6p - 8$ (e) $-15xy^2z^5$

4. (a) binominal (b) monomial (c) trinomial

(d) monomial (e) trinomial (f) binomial

6. (a) 3 (b) 2 (c) 4 (d) 5 (e) 1

7. (a) $5x^3 - 4x^2 + 3x + 18$ (b) $-3x^2 + 4x + 8$ (c) $-3m^2 - 2m + 6$

(d) $y^3 - 3y^2 + y + 1$ (e) $4a^2 - 3a + 5$

Exercise 4.3

1. (a) 15x (b) 2y (c) $-4m^2$ (d) $3a - 4b + 6p$

2. (a) $5a + 4b$ (b) $12x + 3$ (c) $m - 2n + 3p + 4q$

(d) $2a + 4b - 3c$ (e) $x^2 + 6x + 5$ (f) $-7m - 1$

(g) $14p - 8q$ (h) $5a - 3b + 7c + 2x + y - z$

Exercise 4.4

1. (a) $3a + 3b$ (b) $x + 4y$ (c) $2m + 11$ (d) $3p + 5q - r + 2$

(e) $x^2 + 8x + 2$ (f) $a + b + 7c$ (g) $x - 3y + 19z$

2. $2x - y - 2z$ 3. $-2a - b + c$

Exercise 4.5

1. x^7 2. a^{10} 3. $60p^{16}$ 4. $84y^3$ 5. $45c^{10}$ 6. $-77x^6$

7. $-32z^5$ 8. $-24m^7$ 9. $12n^8$ 10. $90x^{20}$

Exercise 4.6

1. (a) a^5 (b) m^6 (c) $-x^6$ (d) $-y^9$ (e) 2p (f) $-4c^{11}$

2. (a) $4n^2$ (b) 3 c (c) $-p^4$ (d) $-5c^6$ (e) $2x^{12}$

Exercise 4.7

1. (a) 3 (b) 7 (c) 7 (d) 3 (e) -1 (f) 5

2. (a) 2 (b) -7 (c) 5 (d) -1 (e) 4

3. 36 sq. m , 24 metre 4. 42 sq. cm

Exercise 4.8

1. (a) $x+7=10$ (c) $2p=6$ (e) $\frac{K}{3}=5$ (g) $2y-7=25$

2. (a) $x+12=27$ (b) $5x+1=16$ (c) $8-y=6$ (d) $2x+5=25$

   (e) $7p=42$ (f) $\frac{z}{3}=2$ (g) $x-20=7$ (h) $6x=72$

   (i) $9+x=42$ (j) $55+x=95$

Exercise 4.9

1. (a) 2 (b) 3 (c) 2 (d) 4 (e) 1
2. (a) 16 (b) – 28 (c) 5 (d) 24 (e) 3
   (f) 18 (g) 9 (h) – 3 (i) 4 (j) – 2
3. 15 4. 30 5. 15 6. 88 7. 5

Unit Test – 4

Part I

1. (c) 5 2. (b) monomial 3. (d) 5 x 4. (a) $2x^2$ 5. (b) 3 6. – 4 x
7. a^8 8. Degree 9. 1 10. x^3 11. Added
12. Simple 13. 3 14. Trinomial

Part II

15. $10x^2$ 16. x^3+2x^2-3x-7 18. $x+4y$ 19. $-4x^5$
20. 11 21. $x+17=30$ 22. 21

Part III

23. $2x^2-2x+4$ 24. $-4y^7$ 25. 6 26. $15a^2-2a-4$

Exercise 5.2

1. (b) 1 1 (c) 0 6 (d) 0 4 (e) 1 1 (f) 1 1
2. (b) 1 1 (c) 4 4 (d) 2 2 (e) 0 2
3. (a) 3 3 (b) 4 4 (c) 2 2

Exercise 5.3

2. Equilateral triangles, squares and hexagons
3. 60° , 90° , and 120°

Exercise 5.5

1. (a) $\lfloor POS$ and $\lfloor SOR$; $\lfloor POQ$ and $\lfloor QOR$

   (b) $\lfloor POR$

2. (a) 60° (b) 65° (c) 20° (d) 30° (e) 30° (f) 80°

3. $x=60°, y=120°$ 4. $\lfloor AOD = \lfloor DOB = \lfloor BOC = 90°$

Exercise 5.6

1. (a) 100° (b) 80° (c) 80°
2. x = 105° , y = 75°
3. $\angle BOX = 112°$, $\angle BOP = 68°$, $\angle AOP = 112°$, $\angle OPC = 68°$, $\angle OPD = 112°$, $\angle CPY = 112°$, $\angle DPY = 68°$

Exercise 5.7

1. x = 45°
2. $\angle P = 90°$, $\angle Q = 60°$, $\angle R = 30°$
3. 50° , 80°
4. $\angle ACB = 80°$; $\angle ACD = 100°$
5. 60°

Unit Test – 5

1.(a) 4 2. (d) 3 3. (c) a direction and a distance

4.(b)

5. (d) 360° 6. (b) 180° 7. Perpendicular bisector 8. Mirror symmetry
9. Two 10. Six 11. No Symmetry 12. Rotational
13. No 14. Equal 15. 180°
16. Mirror symmetry and rotational symmetry 18. (a) 5 sides (b) 6 sides
19. Squares , triangles and hexagons

22.

23. 60°
24. $\angle AOB$ & $\angle BOC$ or $\angle AOD$ & $\angle COD$
29. $\angle ROP = 105°$, $\angle POS = 75°$ and $\angle QOS = 105°$
30. 114° , 57°

Exercise 7.1

2. (a) 20 (b) 90 (c) 10 (d) 40 – 50 (e) 60 – 70