S·C·R·A–2009

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T.B.C. : Q–TDS–J–NC

Test Booklet Series

Serial No. 125153

A

TEST BOOKLET

MATHEMATICS

Paper—III

Time Allowed : Two Hours

Maximum Marks : 200

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C** OR **D** AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE ANSWER SHEET.
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. *DO NOT* write *anything else* on the Test Booklet.
4. This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.
5. You have to mark all your responses *ONLY* on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet*. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**

    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.

    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third (0·33)** of the marks assigned to that question will be deducted as penalty.

    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.

    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. Let A_1, A_2 and A_3 be subsets of a set X. Which one of the following is correct?

(a) $A_1 \cup A_2 \cup A_3$ is the largest subset of X containing A_1, A_2 and A_3

(b) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing elements of each of A_1, A_2 and A_3

(c) The smallest subset of X containing $A_1 \cup A_2$ and A_3 equals the smallest subset of X containing both A_1 and $A_2 \cup A_3$ only if $A_2 = A_3$

(d) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing either A_1 or $A_2 \cup A_3$ but not both

2. If $T_r = V_{r+1} - V_r - 2$, $r = 1, 2, 3, \ldots$, where V_r is the sum of the first r terms of an AP whose first term is r and common difference is $(2r-1)$, then T_r is always

(a) prime and odd

(b) composite and odd

(c) composite and even

(d) odd but not prime

3. If

$$x = \sum_{n=0}^{\infty} a^n, \; y = \sum_{n=0}^{\infty} b^n, \; z = \sum_{n=0}^{\infty} c^n$$

where a, b and c are in AP, and $|a| < 1$, $|b| < 1$, $|c| < 1$; then x, y and z are in

(a) AP

(b) GP

(c) HP

(d) AGP

4. For what values of x and y, the expression $(|z|^2 + |z-5|^2 - |z-zi|^2)$ where $z = x + iy$, assumes minimum value?

(a) $x = 5$, $y = -3$

(b) $x = -5$, $y = 3$

(c) $x = 3$, $y = -5$

(d) $x = -3$, $y = 5$

5. If the expansion in powers of x of the function

$$\frac{1}{(1-ax)(1-bx)}$$

is $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots$, where $|ax|$, $|bx| < 1$, then what is a_n equal to?

(a) $\frac{a^n - b^n}{b-a}$

(b) $\frac{a^{n+1} - b^{n+1}}{b-a}$

(c) $\frac{b^{n+1} - a^{n+1}}{b-a}$

(d) $\frac{b^n - a^n}{b-a}$

6. What is the number of integral points (integral points mean both coordinates must be integers) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0)?

(a) 105

(b) 133

(c) 190

(d) 233

7. A man has 7 relatives, 4 women and 3 men. His wife also has 7 relatives, 3 women and 4 men. What is the number of ways can they invite 3 women and 3 men so that 3 of them are the man's relatives and 3 of them are his wife's relatives?

(a) 485

(b) 484

(c) 468

(d) 467

8. If $a_1, a_2, \ldots, a_{50}$ are in GP, then what is

$$\frac{a_1 - a_3 + a_5 - \ldots + a_{49}}{a_2 - a_4 + a_6 - \ldots + a_{50}}$$

equal to?

(a) 0

(b) 1

(c) $\frac{a_1}{a_2}$

(d) $\frac{a_{25}}{a_{24}}$

9. Let U be a set with number of elements in U is 2009.

Consider the following statements :

1. If A, B are subsets of U with $n(A \cup B) = 280$, then

$$n(A' \cap B') = x_1^3 + x_2^3 = y_1^3 + y_2^3$$

for some positive integers x_1, x_2, y_1, y_2.

2. If A is a subset of U with $n(A) = 1681$ and out of these 1681 elements, exactly 1075 elements belong to a subset B of U, then $n(A - B) = m^2 + p_1 p_2 p_3$ for some positive integer m and distinct primes p_1, p_2, p_3.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

10. What is the probability that three randomly selected persons will be born on different days of the week?

(a) $\frac{36}{49}$

(b) $\frac{30}{49}$

(c) $\frac{25}{49}$

(d) $\frac{19}{49}$

11. A measure of central tendency $M(x_1, x_2, \dots, x_n)$ of n discrete observations $x_1, x_2, \dots, x_n$ satisfies the property P, if

$$M(x_1 + a, x_2 + a, \dots, x_n + a) = M(x_1, x_2, \dots, x_n) + a$$

for every a.

Some measures of central tendency are given below :

1. Arithmetic mean
2. Geometric mean
3. Harmonic mean
4. Median

Which of the above measures satisfies/satisfy the property P?

(a) 1 only

(b) 1 and 4

(c) 1, 2 and 3

(d) 4 only

12. Let $n \in N$ be such that $n \geq 1$. Let $A = \{1, 2, \dots, n\}$. Let $f : P(A) \to \{0, 1\}$ be defined by

$$f(B) = 0, \text{ if } 1 \notin B$$
$$= 1, \text{ if } 1 \in B$$

Consider the following statements :

1. f can never be a bijection. That is there exists no $n \in N$ for which f is a bijection.
2. There exists at least one $n \in N$ for which f is not onto.
3. If n is a prime number, then f will be one-to-one.

Which of the statements given above are ***not*** correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

13. Let $A = N \times N$ be the Cartesian product of N and N. Let

$$S = \{((m, n), (p, q)) \in A \times A : m + q = n + p\}$$

Consider the following statements :

1. If $((m, n), (p, q)) \in S$ and $((p, q), (r, s)) \in S$, then $((r, s), (m, n)) \in S$.
2. There exists at least one element $((m, n), (p, q)) \in S$ such that $((p, q), (m, n)) \notin S$.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

14. Let M be a set of all 2×2 matrices over a set of rational numbers such that for any $A, B \in M$, the operation $*$ is defined as

$$A * B = \frac{AB + BA}{2}$$

Which one of the following is ***not*** correct?

(a) $*$ is associative

(b) $*$ is commutative

(c) $*$ is binary operation on M

(d) M is closed under $*$

15. Let $A = \{1, 2, 3\}$. Let $B = P(A)$, the power set of A.

Consider the following statements :

1. The number of binary operations that can be defined on B is 8^{16}.
2. The number of binary operations that can be defined on $A \times A$ is 9^{81}.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

16. If

$$\sin^2 \theta = \frac{x^2 + y^2}{2xy}, \quad x, y \in R$$

then which one of the following is correct?

(a) $x \neq y$

(b) $x = y \neq 0$

(c) $x \neq y \neq 0$

(d) $x > 0$, y may be zero

17. What is the value of

$$\sum_{k=0}^{100} k i^k$$

where $i^2 = -1$?

(a) $50 - 50i$

(b) $50 - 49i$

(c) $49 - 50i$

(d) $51 - 50i$

18. Let ω be complex (but not real) cubic root of unity. Which one of the following is correct?

(a) $\overline{\omega}$ is of absolute value greater than 1

(b) $-\omega$ will be a cubic root of unity

(c) The absolute value of $(-1 - \omega)$ is 1

(d) $(\overline{\omega})^6 = 1$ but $(\overline{\omega})^3 \neq 1$

19. Let n be a positive integer whose representation in the binary form is given by $(y_1\ y_2\ y_3\ \dots\ y_k)_2$, where k is even. Suppose

$$y_1 = y_3 = \dots = y_{k-1} = 1$$

$$y_2 = y_4 = \dots = y_k = 0$$

Which one of the following is correct?

(a) $n = \dfrac{2^k - 2}{3}$

(b) $n = \dfrac{2^{k+1} - 2}{3}$

(c) $n = \dfrac{2^{\frac{k}{2}} - 1}{3}$

(d) $n = \dfrac{2^{\frac{k}{2}} - 2}{3}$

20. What is the number of positive integral solutions of $x_1 x_2 x_3 = 30$?

(a) 25

(b) 26

(c) 27

(d) 28

21. If a, b and c are in GP such that $a + b + c = xb$, then which one of the following is correct?

(a) $x < -1$ or $x > 3$

(b) $x < -1$ and $x > 3$

(c) $x > -1$ or $x < 3$

(d) $x > -1$ and $x < 3$

22. Which term of the sequence $9 - 8i$, $8 - 6i$, $7 - 4i$ is purely imaginary?

(a) 5th

(b) 9th

(c) 10th

(d) None of the above

23. If

$$\sum_{i=1}^{n} t_i = \frac{(n+2)(n+1)n}{6}$$

then what is $\sum_{i=1}^{n} \frac{1}{t_i}$ equal to?

(a) $\dfrac{2n}{n+1}$

(b) $\dfrac{2n}{n+2}$

(c) $\dfrac{n}{n+1}$

(d) $\dfrac{n}{n+2}$

24. The roots of the equation

$$(a^4 + b^4)x^2 + (4abcd)x + (c^4 + d^4) = 0$$

are given to be real. Then which one of the following is correct?

(a) The roots are always different

(b) The roots are always identical

(c) The roots may be different or identical

(d) No conclusion can be drawn

25. What is the remainder when $(27^{10} + 7^{51})$ is divided by 10?

(a) 1 (b) 2

(c) 3 (d) 5

26. What is the coefficient of x^5 in the expansion of

$$(1+x)^{21} + (1+x)^{22} + (1+x)^{23} + \dots + (1+x)^{30}\ ?$$

(a) $C(51, 5)$

(b) $C(31, 5) - C(21, 5)$

(c) $C(31, 6) - C(21, 6)$

(d) $C(31, 6) + C(21, 6)$

27. Consider

$$G = \left\{ \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} : \theta \in R \right\}$$

For any $\theta \in R$, denote the matrix

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

by A_θ.

Consider the following statements :

1. $\text{Adj}(A_\theta) = -A_\theta$ for some $\theta \in R$, then $\theta = \frac{(2r+1)\pi}{2}$ for some integer r.
2. $\text{Adj}(A_\theta) = A_\theta^T$ holds for only finitely many $\theta \in R$.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

28. Let p be any prime > 2 and $S = \{2, 4, 6, \dots, 2(p-1)\}$. Define the operation $(p \odot_n q)$ as remainder when pq is divided by n, where n is a natural number and $q \in S$. What is the identity element of $(S, \odot_{2p})$?

(a) $p - 1$

(b) $p + 1$

(c) $p + 3$

(d) None of the above

29. If $\cos(\theta - \alpha) = a$ and $\cos(\theta - \beta) = b$, then what is the value of

$$\sin^2(\alpha - \beta) + 2ab\cos(\alpha - \beta)?$$

(a) $a^2 + b^2$

(b) $a^2 - b^2$

(c) $-a^2 - b^2$

(d) $-a^2 + b^2$

30. Let $\frac{\pi}{2} < \theta < \pi$ and $\pi < \phi < \frac{3\pi}{2}$. If $\sin\theta + \sin(\theta+\phi) + \sin(\theta-\phi) = 0$, then which one of the following is correct?

(a) $\sin^2\phi\cot^2\theta$ is an integer

(b) $\text{cosec}^2\phi\tan^2\theta$ is an integer

(c) $\sin^2\phi\cot^2\theta$ is irrational

(d) $\text{cosec}\,\phi\tan\theta < -3$

31. If $\sin x + \sin y = 3(\cos y - \cos x)$, then what is the value of

$$\sin 3x \text{ cosec } 3y?$$

(a) –1

(b) 0

(c) 1

(d) None of the above

32. What is the image of

$$\left[0, \frac{\sqrt{3}-1}{2\sqrt{2}}\right]$$

under the inverse sine function?

(a) $\left[0, \frac{\pi}{10}\right]$

(b) $\left[0, \frac{\pi}{8}\right]$

(c) $\left[0, \frac{\pi}{15}\right]$

(d) $\left[0, \frac{\pi}{12}\right]$

33. The real value of x such that $e^{\sin x} - e^{-\sin x} = 4$

(a) is $\frac{3\pi}{4}$

(b) is $\frac{\pi}{3}$

(c) is $\frac{\pi}{6}$

(d) does not exist

34. AB is a vertical pole. The end A is on the level of ground. C is the middle point of AB. P is a point on the level of ground. The portion BC subtends an angle β at P. If $AP = n(AB)$, then what is $\tan\beta$ equal to?

(a) $\frac{n}{2n^2+1}$ (b) $\frac{n}{n^2-1}$

(c) $\frac{n}{n^2+1}$ (d) $\frac{n}{2n^2-1}$

35. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

If U_1, U_2 and U_3 are column matrices satisfying

$$AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

and U is 3×3 matrix whose columns are U_1, U_2 and U_3, then what is $|U|$ equal to?

(a) –3 (b) $\frac{3}{2}$

(c) 2 (d) 3

36. If

$$A = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

then what is $|A^{1003} - 5A^{1002}|$ equal to?

(a) −5 *(b)* 1

(c) 8 *(d)* 21

37. If

$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

and $a^2 + b^2 + c^2 = -2$, then what is the degree of the polynomial $f(x)$?

(a) 0

(b) 1

(c) 2

(d) 3

38. Let

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$$

and $(10)B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$

If B is the inverse of matrix A, then what is the value of α?

(a) −2 *(b)* −1

(c) 2 *(d)* 5

39. Let ABC be an equilateral triangle. If R is the circumradius and r is the inradius of triangle ABC, then what is $\sqrt{R^2 - r^2}$ equal to?

(a) $\sqrt{5}r$

(b) $\sqrt{7}r$

(c) $\sqrt{3}r$

(d) $2r$

40. ABC is a triangle in which $BC = a$, $CA = b$ and $AB = c$. Let Δ denote the area of triangle ABC.

Consider the following statements :

1. $\Delta > \min\left(\frac{ab}{2}, \frac{bc}{2}, \frac{ca}{2}\right)$
   (min stands for minimum)

2. If $\Delta = \min\left(\frac{ab}{2}, \frac{bc}{2}, \frac{ca}{2}\right)$, then ABC is a right-angled triangle.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

41. Let $0° < \theta < 90°$. Let $\alpha = \sec^2\theta$, $\beta = \tan^2\theta$.

Consider the following statements :

1. $(\alpha^2 - \beta^2)^2 > (\alpha^3 - \beta^3)$
2. There exists a natural number n such that $(\alpha + \beta)^n$ cannot be expressed as a polynomial expression in β with integral coefficients.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

42. Let A, B be the acute angles such that $\cos A = \frac{n+1}{\sqrt{(n+1)^2 + n^2}}$ and $\cos B = \frac{2n+1}{\sqrt{(2n+1)^2 + 1}}$ for some positive integer n. Which one of the following is correct?

(a) $(A + B) > 90°$

(b) $60° < (A + B) < 90°$

(c) $15° < (A + B) < 30°$

(d) $30° < (A + B) < 60°$

43. Let P_1T_1 and P_2T_2 denote two towers of equal height h; T_1 and T_2 denote respectively their tops; P_1 and P_2 denote respectively their bases. Let A be a point on the line segment joining P_1 and P_2 such that $P_1A = x$ and $AP_2 = y$. If α is the angle of elevation of T_1 as seen from A and β is the angle of elevation of T_2 as seen from A, then consider the following statements :

1. $\frac{x}{y} = \tan\alpha \cot\beta$
2. If $(\alpha + \beta)$ is acute, then $xy > h^2$

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

44. If

$$f(x) = \lim_{n \to \infty} \frac{\ln(2 + x) - x^{2n} \sin x}{1 + x^{2n}}$$

then what is $\lim_{x \to 1} f(x)$ equal to?

(a) $\ln 3$

(b) $-\sin 1$

(c) 0

(d) Limit does not exist

45. Consider the following statements :

The derivative of $\sin^{-1}(3x - 4x^3)$ is

1. $\frac{3}{\sqrt{1-x^2}}$ for all $x \in R$
2. $\frac{3}{\sqrt{1-x^2}}$ for $|x| < \frac{1}{2}$
3. $\frac{-3}{\sqrt{1-x^2}}$ for $x \in \left(-1, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, 1\right)$

Which of the statements given above is/are correct?

(a) 1

(b) 2 only

(c) 3 only

(d) Both 2 and 3

46. If $A_i = \frac{x - a_i}{|x - a_i|}$, $i = 1, 2, 3, \ldots, n$ and $a_1 < a_2 < a_3 < \ldots < a_n$, then

$$\lim_{x \to a_m} (A_1 A_2 A_3 \ldots A_n)$$

where $1 \le m \le n$

(a) is equal to $(-1)^m$

(b) is equal to $(-1)^{m+1}$

(c) is equal to $(-1)^{n-m}$

(d) does not exist

47. If x and y are the sides of two squares such that $y = x - x^2$, then what is the rate of change of the area of second square with respect to the area of first square?

(a) $(1 - x^2)x$

(b) $2(1 - x^2)x$

(c) $2x^2 - 3x + 1$

(d) $2(2x^2 - 3x + 1)$

48. If $[x]$ denotes the greatest integer function, then what is

$$\int_1^3 \frac{dx}{x^2 + [x]^2 + 1 - 2x[x]}$$

equal to?

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{2}$

(d) 1

49. If $u = \cot x$ and $I_n = \int u^n \, dx$, then what is

$I_0 + I_1 + 2(I_2 + I_3 + \ldots + I_8) + I_9 + I_{10}$

equal to?

(a) $u + \frac{u^2}{2} + \ldots + \frac{u^9}{9}$

(b) $-\left(u + \frac{u^2}{2} + \ldots + \frac{u^9}{9}\right)$

(c) $-\left(u + \frac{u^2}{2!} + \ldots + \frac{u^9}{9!}\right)$

(d) $\frac{u}{2} + \frac{2u^2}{3} + \ldots + \frac{9u^9}{10}$

50. If $f(x)$ is continuous at $x = 0$ and $f(0) = 2$, then what is the value of

$$\lim_{x \to 0} \left[\frac{\int_0^x f(u)\, du}{x}\right]?$$

(a) 0

(b) 1

(c) 2

(d) $f(2)$

51. If

$$f(x) = \frac{x}{2} - 1$$

then on the interval $[0, \pi]$

(a) $\tan(f(x))$ and $\frac{1}{f(x)}$ are both continuous

(b) $\tan(f(x))$ and $\frac{1}{f(x)}$ are both discontinuous

(c) $\tan(f(x))$ and $f^{-1}(x)$ are both continuous

(d) neither $\tan(f(x))$ nor $f^{-1}(x)$ is continuous

52. If

$$F(x) = \frac{1}{x^2}\int_4^x [4t^2 - 2F'(t)]\, dt$$

then what is $F'(4)$ equal to?

(a) $\frac{32}{9}$

(b) $\frac{64}{3}$

(c) $\frac{64}{9}$

(d) None of the above

53. If

$$y = \int_0^x f(t)\sin[k(x-t)]\, dt$$

then what is $\frac{d^2y}{dx^2} + k^2 y$ equal to?

(a) 0 *(b)* y

(c) $kf(x)$ *(d)* $k^2 f(x)$

54. If $\phi(a-x) = \phi(x)$, then what is

$$\int_0^a x\,\phi(x)\, dx$$

equal to?

(a) $a\int_0^a \phi(x)\, dx$

(b) $x\int_0^a \phi(x)\, dx$

(c) $2a\int_0^{\frac{a}{2}} \phi(x)\, dx$

(d) $\frac{a}{2}\int_0^a \phi(x)\, dx$

55. The general solution of the differential equation

$$\frac{dy}{dx} + y\frac{d\phi}{dx} = \phi(x)$$

where ϕ is a function of x alone is given by

(a) $y = \phi + ce^{-\phi}$

(b) $y = \phi + 1 - ce^{-\phi}$

(c) $y = \phi - 1 + ce^{\phi}$

(d) $y = \phi - 1 + ce^{-\phi}$

where c is a constant.

56. If $[x]$ and $\{x\}$ represent integer and fractional parts of x respectively, then what is the expression

$$[x] + \sum_{r=1}^{2000} (\{x+r\}/2000)$$

equal to?

(a) $\frac{2001}{2}x$

(b) $x + 2001$

(c) x

(d) $[x] + \frac{2001}{2}$

57. If $A > 0$, $B > 0$ and $A + B = \frac{\pi}{3}$, then what is the maximum value of $\tan A \tan B$?

(a) 3 (b) $\frac{1}{3}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

58. Let f be a positive function. Let

$$I_1 = \int_{1-k}^{k} x f\{x(1-x)\}\, dx$$

$$I_2 = \int_{1-k}^{k} f\{x(1-x)\}\, dx$$

where $(2k-1) > 0$, then what is $\frac{I_1}{I_2}$ equal to?

(a) 2 (b) k

(c) $\frac{1}{2}$ (d) 1

59. A point moves along the curve $y = (\sin 2x) + 1$, $-2\pi \le x \le 2\pi$. How many times does the point attain the maximum height from the x-axis?

(a) 8 (b) 4

(c) 2 (d) 1

60. The equation

$$a_0 x^n + a_1 x^{n-1} + \ldots + a_n = 0$$

has at least one root between 0 and 1, if

(a) $\frac{a_0}{n} + \frac{a_1}{n-1} + \ldots + a_{n-1} = 0$

(b) $\frac{a_0}{n-1} + \frac{a_1}{n-2} + \ldots + a_{n-2} = 0$

(c) $na_0 + (n-1)a_1 + \ldots + a_{n-1} = 0$

(d) $\frac{a_0}{n+1} + \frac{a_1}{n} + \ldots + a_n = 0$

61. If

$$2f(x) - 3f\left(\frac{1}{x}\right) = x^2, \; x \ne 0$$

then what is $f(2)$ equal to?

(a) $\frac{3}{4}$

(b) $-\frac{3}{4}$

(c) $\frac{5}{4}$

(d) $-\frac{7}{4}$

62. If the sets $\{(t, t) \mid t \in R\}$ and $\{(t, t^2) \mid t \in R\}$ represent the graphs of two functions f and g on R respectively, then what is the nature of the graph of $(f + g)$?

(a) Straight line passing through origin

(b) A parabola through origin

(c) A parabola through origin symmetrical about y-axis

(d) A parabola through origin symmetrical about x-axis

63. The differential equation

$$y'' + ky' + 4y = 0$$

has solution of the form

$$y = Ae^{ax} \cos bx + Be^{ax} \sin bx$$

for all values of k, if

(a) $-4 < k < 4$

(b) $k < -4$, $k > 4$

(c) $k = 0$ or 4

(d) None of the above

64. Let f be twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$, $h(x) = \{f(x)\}^2 + \{g(x)\}^2$. If $h(5) = 11$, then what is $h(10)$ equal to?

(a) 22

(b) 11

(c) 0

(d) Cannot be determined due to insufficient data

65. It is given that three distinct points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear. Then a necessary and sufficient condition for (x_2, y_2) to lie on the line segment joining (x_3, y_3) to (x_1, y_1) is

(a) either $(x_1 + y_1) < (x_2 + y_2) < (x_3 + y_3)$ or $(x_1 + y_1) > (x_2 + y_2) > (x_3 + y_3)$

(b) either $(x_1 - y_1) < (x_2 - y_2) < (x_3 - y_3)$ or $(x_1 - y_1) > (x_2 - y_2) > (x_3 - y_3)$

(c) either $0 < \left(\frac{x_2 - x_3}{x_1 - x_3}\right) < 1$ or $0 < \left(\frac{y_2 - y_3}{y_1 - y_3}\right) < 1$

(d) None of the above

66. The difference between the radii of the largest and the smallest circles, which have their centres on the circumference of the circle

$$x^2 + y^2 + 2x + 4y = 4$$

and pass through the point (a, b) lying outside the given circle, is

(a) 1

(b) 2

(c) 3

(d) 6

67. All points whose distance from the nearest point on the circle $(x-1)^2 + y^2 = 1$ is half the distance from the line $x = 5$ lie on

(a) an ellipse

(b) a pair of straight lines

(c) a parabola

(d) a circle

68. The locus of a moving point $P(x, y)$ in the plane satisfies the equation

$$2x^2 = r^2 + r^4$$

where $r^2 = x^2 + y^2$. Which one of the following is correct?

(a) For every positive real number d, there is a point (x, y) on the locus such that $r = d$

(b) For every value of d, $0 < d < 1$, there are exactly four points on the locus each of which is at a distance d from the origin

(c) The point P always lies in the first quadrant

(d) None of the above

69. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then

$$|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$$

does ***not*** exceed

(a) 12

(b) 9

(c) 8

(d) 6

70. The vectors $2\hat{i}+\log_3 x\hat{j}+a\hat{k}$ and $-3\hat{i}+a\log_3 x\hat{j}+\log_3 x\hat{k}$, where $x>3$ are inclined at an angle for

(a) $a=0$

(b) $a<0$

(c) $a>0$

(d) No real value of a

71. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vectors such that $\vec{a}+\vec{b}+\vec{c}=\alpha\vec{d}$ and $\vec{b}+\vec{c}+\vec{d}=\beta\vec{a}$, then what is $\vec{a}+\vec{b}+\vec{c}+\vec{d}$ equal to?

(a) $\vec{0}$

(b) $\alpha\vec{a}$

(c) $\beta\vec{b}$

(d) $(\alpha+\beta)\vec{c}$

72. The vector $\vec{a}$ coplanar with the vectors $\hat{i}$ and $\hat{j}$, perpendicular to the vector $\vec{b}=4\hat{i}-3\hat{j}+5\hat{k}$ such that $|\vec{a}|=|\vec{b}|$, is

(a) $\pm\sqrt{2}(3\hat{i}+4\hat{j})$

(b) $\pm\sqrt{2}(4\hat{i}+3\hat{j})$

(c) $\pm\sqrt{2}(4\hat{i}+5\hat{j})$

(d) $\pm\sqrt{2}(5\hat{i}+4\hat{j})$

73. One end of a focal chord of the parabola $y^2=8ax$ is the point $(2at^2, 4at)$. What is the other end of the chord?

(a) $\left(\frac{a}{t^2}, \frac{-2\sqrt{2}a}{t}\right)$

(b) $\left(\frac{a}{2t^2}, \frac{-2a}{t}\right)$

(c) $\left(\frac{2a}{t^2}, \frac{-4a}{t}\right)$

(d) $\left(\frac{2a}{t^2}, \frac{4a}{t}\right)$

74. A point P lies on the line passing through the points (12, 3, 3) and (–3, 0, 9). If the z-coordinate of P is 1, what are the x and y coordinates?

(a) $x=4,\ y=17$

(b) $x=7,\ y=2$

(c) $x=2,\ y=7$

(d) $x=17,\ y=4$

75. The position vectors of two points P and Q are $3\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}-2\hat{j}-4\hat{k}$ respectively. The position vector of the middle point of the segment PQ makes an angle ϕ with the position vector of Q. What is ϕ equal to?

(a) $\cos^{-1}\frac{2}{3}$

(b) $\cos^{-1}\frac{1}{3}$

(c) $\cos^{-1}\frac{1}{\sqrt{3}}$

(d) $30°$

76. If $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ and $\vec{\delta}$ be four vectors such that $\vec{\delta} = (\vec{\alpha} \cdot \vec{\gamma})\vec{\beta} - (\vec{\alpha} \cdot \vec{\beta})\vec{\gamma}$, then which one of the following is correct?

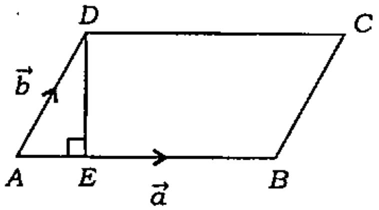
(a) $\vec{\alpha}$ is parallel to $\vec{\delta}$

(b) $\vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \vec{\gamma}$

(c) $\vec{\alpha}$ is perpendicular to $\vec{\delta}$

(d) None of the above

77.



What is $\overrightarrow{ED}$ in the above diagram?

(a) $\dfrac{|\vec{a}|^2 \vec{b} - (\vec{a} \cdot \vec{b})\vec{a}}{|\vec{a}|^2}$

(b) $\dfrac{|\vec{a}|^2 \vec{b} + (\vec{a} \cdot \vec{b})\vec{a}}{|\vec{a}|^2}$

(c) $\dfrac{-|\vec{a}|^2 \vec{b} + (\vec{a} \cdot \vec{b})\vec{a}}{|\vec{a}|^2}$

(d) $\dfrac{-|\vec{a}|^2 \vec{b} - (\vec{a} \cdot \vec{b})\vec{a}}{|\vec{a}|^2}$

where $\vec{a}$ and $\vec{b}$ are the vectors representing the adjacent sides of parallelogram $ABCD$ as shown in the figure.

78. Let $\vec{r}$ be the position vector of a point $P(x, y, z)$, where x, y and z are natural numbers and $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. What is the total number of possible positions of point P for which $\vec{r} \cdot \vec{a} = 10$?

(a) 18

(b) 36

(c) 66

(d) 72

79. If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors and $\vec{r}$ is a vector satisfying

$$\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = [\vec{r} \;\; \vec{a} \;\; \vec{b}] = 1$$

then what is $\vec{r}$ equal to?

(a) $(\vec{a} \times \vec{b}) + \vec{a} + \vec{b}$

(b) $(\vec{a} \times \vec{b}) - \vec{a} + \vec{b}$

(c) $(\vec{a} \times \vec{b}) - (\vec{a} + \vec{b})$

(d) $(\vec{a} \times \vec{b}) + \vec{a} - \vec{b}$

80. A ray of light coming from origin after reflection at a point $P(x, y)$ of a curve becomes parallel to x-axis. What may be the differential equation of the curve?

(a) $y\left(\dfrac{dy}{dx}\right)^2 + 2x\left(\dfrac{dy}{dx}\right) + y = 0$

(b) $y\left(\dfrac{dy}{dx}\right)^2 + 2x\left(\dfrac{dy}{dx}\right) - y = 0$

(c) $y\left(\dfrac{dy}{dx}\right)^2 - 2x\left(\dfrac{dy}{dx}\right) + y = 0$

(d) $y\left(\dfrac{dy}{dx}\right)^2 + 2x\left(\dfrac{dy}{dx}\right) = 0$

81. What is the general solution of the differential equation

$$\frac{dy}{dx} + y\frac{dh(x)}{dx} = h(x)\frac{dh(x)}{dx}$$

where $h(x)$ is given function of x?

(a) $h(x) + \ln[1 + y - h(x)] = c$

(b) $h(x) + \ln[1 + y + h(x)] = c$

(c) $h(x) + \ln[y - h(x)] = c$

(d) $h(x) + \ln[1 - y - h(x)] = c$

where c is a constant.

82. A curve $y = f(x)$ passes through origin O and lies in the first quadrant. Let PQ, PR be the perpendiculars drawn on x, y axes respectively from the point $P(x, y)$ such that the area of the region OPR is twice that of region OPQ. What is the equation of the curve?

(a) $y = cx^2$

(b) $y^2 = cx$

(c) $y = cx^3$

(d) $y^2 = cx^3$

where c is a constant.

83. What does the equation

$$x^2 - 5x + 6 = 0$$

represent in three-dimensional space?

(a) Skew lines

(b) A pair of non-parallel planes

(c) A pair of parallel planes

(d) None of the above

84. Consider the following statements :

1. Three planes may intersect in a point.
2. Three planes may intersect in a line.
3. Three planes may be such that two of them are parallel and the third one may intersect them in parallel lines.
4. Three planes may not intersect.

How many possibilities are correct?

(a) One

(b) Two

(c) Three

(d) Four

85. Which one of the following statements best describes the relationship between a frequency distribution and the data set it represents?

(a) The frequency distribution is a simpler but completely equivalent representation of the parent data

(b) The frequency distribution is constructed to identify errors in the original data

(c) In a frequency distribution, values in certain ranges are clubbed together at the midrange value, which makes the representation easy to comprehend

(d) The frequency distribution is simple to understand but not useful for any computation of practical importance

86. The statements below relate to the histogram of a frequency distribution. Which one of the following is ***not*** correct?

(a) The area included in the histogram represents the total frequency of the frequency distribution

(b) If all class intervals are equal, the width of the tallest rectangular bar of the histogram represents the modal class

(c) If all class intervals are equal, the midpoint of the width of the tallest rectangular bar is the mode of the distribution

(d) The height of the histogram at any point represents the frequency per unit interval for the relevant interval

87. A symmetric coin is tossed until the first head is observed. What is the probability that more than seven tosses will be required?

(a) $\frac{1}{7}$

(b) $\frac{1}{49}$

(c) $\frac{1}{64}$

(d) $\frac{1}{128}$

88. A wall measures 40 m by 30 m and contains a window of size 20 m by 20 m. The wall is hit by four stones thrown up by a mower. Assuming that each stone hits the wall in a random position independently of other stones, what is the probability that at least one throw would hit the window?

(a) $\frac{16}{81}$

(b) $\frac{1}{81}$

(c) $\frac{65}{81}$

(d) $\frac{1}{256}$

89. A number is selected at random from a set of first 100 natural numbers. What is the probability that it will be divisible neither by 5 nor by 6?

(a) 0·67

(b) 0·54

(c) 0·33

(d) 0·16

90. The arithmetic mean and variance of 20 numbers were computed as 12 and 9 respectively. It was then found on scrutiny that a number 8 has been misread as 18. What is the correct variance?

(a) 7·75

(b) 8·50

(c) 8·60

(d) This cannot be computed from the given data

91. If one of the regression coefficients $b_{xy} = 0 \cdot 8$, what are the limits of other regression coefficient b_{yx}?

(a) $0 < b_{yx} \leq 0 \cdot 8$

(b) $0 \leq b_{yx} < 1 \cdot 25$

(c) $0 \leq b_{yx} \leq 1 \cdot 25$

(d) $0 < b_{yx} \leq 1 \cdot 25$

92. The marks secured by two students *A* and *B* in six subjects are given below :

A	36	28	24	45	27	22
B	28	37	42	27	19	26

Which one of the following statements is correct?

(a) The average scores of *A* and *B* are same but *A* is consistent

(b) The average scores of *A* and *B* are not same but *A* is consistent

(c) The average scores of *A* and *B* are same but *B* is consistent

(d) The average scores of *A* and *B* are not same but *B* is consistent

93. Two persons *A* and *B* alternately toss a die, *A* starting the process. He who throws a six score first wins the game. What is the probability that *A* wins the game?

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\frac{4}{7}$

(d) $\frac{6}{11}$

94. What is the equation of plane containing the lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \lambda\vec{d}$?

(a) $[\vec{r}\ \vec{a}\ \vec{c}] = 0$

(b) $[\vec{r}\ \vec{a}\ \vec{c}] = 1$

(c) $[\vec{r}\ \vec{a}\ \vec{c}] = \vec{a} \cdot \vec{c}$

(d) None of the above

95. A straight line $\vec{r} = \vec{a} + \lambda\vec{b}$ meets the plane $\vec{r} \cdot \vec{n} = 0$ in *P*. What is the position vector of *P*?

(a) $\vec{r} = \vec{a} - \frac{\vec{a} \cdot \vec{n}}{\vec{b} \cdot \vec{n}}\vec{b}$

(b) $\vec{r} = \vec{a} + \frac{\vec{a} \cdot \vec{n}}{\vec{b} \cdot \vec{n}}\vec{b}$

(c) $\vec{r} = \vec{a} - \frac{\vec{a} \cdot \vec{n}}{\vec{b} \cdot \vec{n}}\vec{a}$

(d) None of the above

Directions :

Each of the following **five (5)** items consists of two statements, one labelled as 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the code given below :

Code :

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is ***not*** the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

96. Assertion (A) :

The range of the inverse tangent function is the complement of S in $\mathbb{R}$ for some subset S of $\mathbb{R}$ such that $S = P \cup Q$, where $P = \{r \in \mathbb{R} : r \leq \theta\}$ and $Q = \{r \in \mathbb{R} : r \geq -\theta\}$ for some $\theta < 0$.

Reason (R) :

The complement in $\mathbb{R}$ of the domain of definition of the inverse cotangent function is the null set.

97. Assertion (A) :

There exists only a finite number of real numbers θ for which

$4(\cos^3 \theta + \sin^3 3\theta) \neq 3(\cos\theta + \sin 3\theta)$

Reason (R) :

There exist exactly two real numbers $\theta \in [0, \pi/2)$ for which

$4(\cos^3 \theta + \sin^3 2\theta) = 3(\cos\theta + \sin 2\theta)$

98. Let m and n be even positive integers with $n < m$. Let n be a multiple of 4 and m leave remainder 2 on division by 4.

Assertion (A) :

The sum of the odd integers between n and m is odd.

Reason (R) :

The number of odd integers between n and m is $\frac{m-n+2}{2}$.

99. Let m and n be integers such that $(m^2 - 4n) > 0$.

Assertion (A) :

If $m < 0$, then there exists exactly one real number k for which the quadratic equations

$x^2 + mx + n = 0$ and $x^2 - 5x + k = 0$

have a common root.

Reason (R) :

If $m > 0$, then there are exactly two real numbers k for which the quadratic equations $x^2 + mx + n = 0$ and $x^2 - 5x + k = 0$ have a common root.

100. Let p be a prime number. Let

$$q = \frac{2p-1}{2p+2}$$

Assertion (A) :

The $(2p-1)$th term and $(2p)$th term of $(1+q)^{4p}$ are equal.

Reason (R) :

If p is odd and $(2p-1)$ is also a prime number, then HCF of $(2p-1)$ and $(2p+2)$ is 1.

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK

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